2017 Summer Math Packet

for students who have completed Precalculus- Entering AP Calculus

Congratulations, you made it through your math class this year! Your fabulous prize will be an even more challenging and interesting math class for next year. Yay!

Here is a packet to do over the summer to keep your math skills sharp, because we want you to be ready for your new math class in the fall. Do the indicated page(s) each week during the summer. You will find dates on the pages.

Complete your summer packet on separate paper, and remember to show all of your work. Do not do the whole packet right away, or you will forget some of the concepts before the fall. Do not leave the packet until the end of the summer, or you will have forgotten some of the concepts.

You have learned how to do everything in this packet at some point during the year, there is nothing new. Use your notes to help you with the packet. If you get completely stuck, then give one of us a call.

Bring the packet with you to your new math class in the fall. You will have a quiz during the first week of class to make sure you have done the packet and are ready for your new math class. Your math teacher might even give you extra credit for your summer math packet. Who doesn't love extra credit?

Have a wonderful and slightly mathematical summer!

The MSA Math Department

Bronwen Williams	Lauren Zachman	Caitlin Harper	Noah Langseth	Aaron Wojahn
651-353-2309	651-353-2305	651-578 7507 ext: 4010	651-353-2319	651-353-2311
bwilliams@mnmsa.org	lzachman@mnmsa.org	charper@mnmsa.org	nlangseth@mnmsa.org	awojahn@mnmsa.org

AP Calculus AB-Summer Packet-2017

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1.2	p17: 5, 7, 9, 16, 17, 19, 22

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1.3	p26: 36, 38, 41-46, 61

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1.4	p33: 21, 27, 30, 32, 33

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1.5	p42: 7, 10-13, 18, 20, 24

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1.6	p50: 19, 21, 26, 28, 30, 32, 36, 38

Exercises and Problems for Section 1.1

Exercises

1. The population of a city, P, in millions, is a function of t, the number of years since 1970, so $P = f(t)$. Explain the meaning of the statement $f(35) = 12$ in terms of the population of this city.

2. The pollutant PCB (polychlorinated biphenyl) affects the thickness of pelican eggs. Thinking of the thickness, T, of the eggs, in mm, as a function of the concentration, P, of PCBs in ppm (parts per million), we have $T = f(P)$. Explain the meaning of $f(200)$ in terms of thickness of pelican eggs and concentration of PCBs.

3. Describe what Figure 1.8 tells you about an assembly line whose productivity is represented as a function of the number of workers on the line.

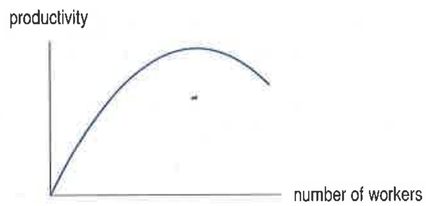


Figure 1.8

For Exercises 4–7, find an equation for the line that passes through the given points.

4. $(0, 0)$ and $(1, 1)$ **5.** $(0, 2)$ and $(2, 3)$

6. $(-2, 1)$ and $(2, 3)$ **7.** $(-1, 0)$ and $(2, 6)$

For Exercises 8–11, determine the slope and the y-intercept of the line whose equation is given.

8. $2y + 5x - 8 = 0$ **9.** $7y + 12x - 2 = 0$

10. $-4y + 2x + 8 = 0$ **11.** $12x = 6y + 4$

12. Match the graphs in Figure 1.9 with the following equations. (Note that the x and y scales may be unequal.)

(a) $y = x - 5$ **(b)** $-3x + 4 = y$
(c) $5 = y$ **(d)** $y = -4x - 5$
(e) $y = x + 6$ **(f)** $y = x/2$

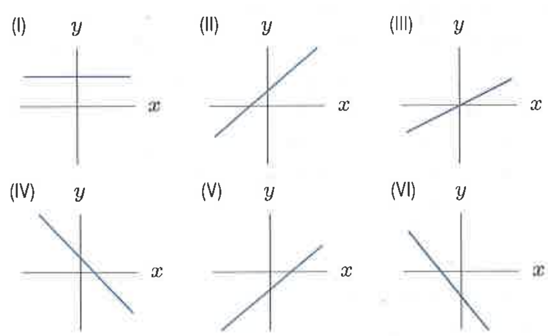


Figure 1.9

13. Match the graphs in Figure 1.10 with the following equations. (Note that the x and y scales may be unequal.)

(a) $y = -2.72x$ **(b)** $y = 0.01 + 0.001x$
(c) $y = 27.9 - 0.1x$ **(d)** $y = 0.1x - 27.9$
(e) $y = -5.7 - 200x$ **(f)** $y = x/3.14$

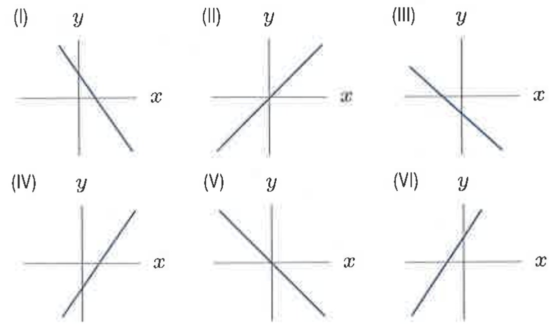


Figure 1.10

14. Estimate the slope and the equation of the line in Figure 1.11.

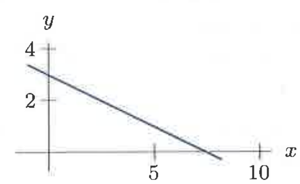


Figure 1.11

15. Find an equation for the line with slope m through the point (a, c).

16. Find a linear function that generates the values in Table 1.3.

Table 1.3

x	5.2	5.3	5.4	5.5	5.6
y	27.8	29.2	30.6	32.0	33.4

For Exercises 17–19, use the facts that parallel lines have equal slopes and that the slopes of perpendicular lines are negative reciprocals of one another.

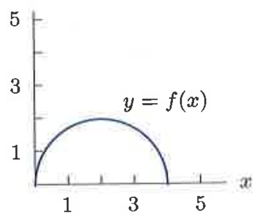
17. Find an equation for the line through the point $(2, 1)$ which is perpendicular to the line $y = 5x - 3$.

18. Find equations for the lines through the point $(1, 5)$ that are parallel to and perpendicular to the line with equation $y + 4x = 7$.

19. Find equations for the lines through the point (a, b) that are parallel and perpendicular to the line $y = mx + c$, assuming $m \neq 0$.

For Exercises 20–23, give the approximate domain and range of each function. Assume the entire graph is shown.

20.



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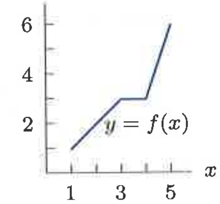

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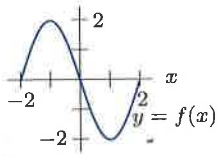

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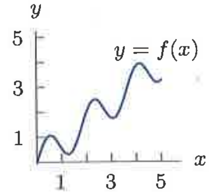


Find domain and range in Exercises 24–25.

24. $y = x^2 + 2$

25. $y = \dfrac{1}{x^2 + 2}$

26. If $f(t) = \sqrt{t^2 - 16}$, find all values of t for which $f(t)$ is a real number. Solve $f(t) = 3$.

In Exercises 27–31, write a formula representing the function.

27. The volume of a sphere is proportional to the cube of its radius, r.

28. The average velocity, v, for a trip over a fixed distance, d, is inversely proportional to the time of travel, t.

29. The strength, S, of a beam is proportional to the square of its thickness, h.

30. The energy, E, expended by a swimming dolphin is proportional to the cube of the speed, v, of the dolphin.

31. The number of animal species, N, of a certain body length, l, is inversely proportional to the square of l.

Problems

In Problems 32–35 the function $S = f(t)$ gives the average annual sea level, S, in meters, in Aberdeen, Scotland,[1] as a function of t, the number of years before 2008. Write a mathematical expression that represents the given statement.

32. In 1983 the average annual sea level in Aberdeen was 7.019 meters.

33. The average annual sea level in Aberdeen in 2008.

34. The average annual sea level in Aberdeen was the same in 1865 and 1911.

35. The average annual sea level in Aberdeen increased by 1 millimeter from 2007 to 2008.

36. In December 2010, the snowfall in Minneapolis was unusually high,[2] leading to the collapse of the roof of the Metrodome. Figure 1.12 gives the snowfall, S, in Minneapolis for December 6–15, 2010.

- **(a)** How do you know that the snowfall data represents a function of date?
- **(b)** Estimate the snowfall on December 12.
- **(c)** On which day was the snowfall more than 10 inches?
- **(d)** During which consecutive two-day interval was the increase in snowfall largest?

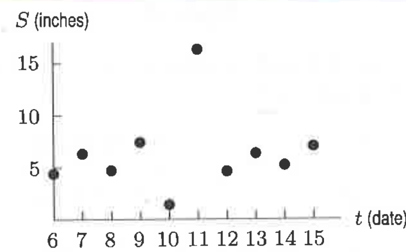


Figure 1.12

37. The value of a car, $V = f(a)$, in thousands of dollars, is a function of the age of the car, a, in years.

- **(a)** Interpret the statement $f(5) = 6$
- **(b)** Sketch a possible graph of V against a. Is f an increasing or decreasing function? Explain.
- **(c)** Explain the significance of the horizontal and vertical intercepts in terms of the value of the car.

38. Which graph in Figure 1.13 best matches each of the following stories?[3] Write a story for the remaining graph.

- **(a)** I had just left home when I realized I had forgotten my books, and so I went back to pick them up.
- **(b)** Things went fine until I had a flat tire.
- **(c)** I started out calmly but sped up when I realized I was going to be late.

[1] www.decc.gov.uk, accessed June 2011
[2] http://www.crh.noaa.gov/mpx/Climate/DisplayRecords.php
[3] Adapted from Jan Terwel, "Real Math in Cooperative Groups in Secondary Education." *Cooperative Learning in Mathematics*, ed. Neal Davidson, p. 234 (Reading: Addison Wesley, 1990).

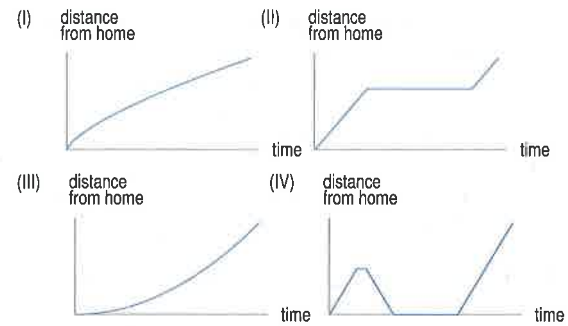


Figure 1.13

39. An object is put outside on a cold day at time $t = 0$. Its temperature, $H = f(t)$, in °C, is graphed in Figure 1.14.

(a) What does the statement $f(30) = 10$ mean in terms of temperature? Include units for 30 and for 10 in your answer.
(b) Explain what the vertical intercept, a, and the horizontal intercept, b, represent in terms of temperature of the object and time outside.

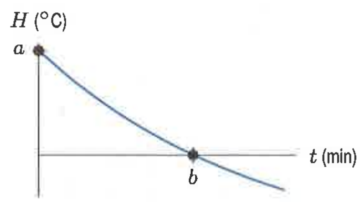


Figure 1.14

40. A rock is dropped from a window and falls to the ground below. The height, s (in meters), of the rock above ground is a function of the time, t (in seconds), since the rock was dropped, so $s = f(t)$.

(a) Sketch a possible graph of s as a function of t.
(b) Explain what the statement $f(7) = 12$ tells us about the rock's fall.
(c) The graph drawn as the answer for part (a) should have a horizontal and vertical intercept. Interpret each intercept in terms of the rock's fall.

41. In a California town, the monthly charge for waste collection is \$8 for 32 gallons of waste and \$12.32 for 68 gallons of waste.

(a) Find a linear formula for the cost, C, of waste collection as a function of the number of gallons of waste, w.
(b) What is the slope of the line found in part (a)? Give units and interpret your answer in terms of the cost of waste collection.
(c) What is the vertical intercept of the line found in part (a)? Give units and interpret your answer in terms of the cost of waste collection.

42. For tax purposes, you may have to report the value of your assets, such as cars or refrigerators. The value you report drops with time. "Straight-line depreciation" assumes that the value is a linear function of time. If a \$950 refrigerator depreciates completely in seven years, find a formula for its value as a function of time.

43. A company rents cars at \$40 a day and 15 cents a mile. Its competitor's cars are \$50 a day and 10 cents a mile.

(a) For each company, give a formula for the cost of renting a car for a day as a function of the distance traveled.
(b) On the same axes, graph both functions.
(c) How should you decide which company is cheaper?

44. Residents of the town of Maple Grove who are connected to the municipal water supply are billed a fixed amount monthly plus a charge for each cubic foot of water used. A household using 1000 cubic feet was billed \$40, while one using 1600 cubic feet was billed \$55.

(a) What is the charge per cubic foot?
(b) Write an equation for the total cost of a resident's water as a function of cubic feet of water used.
(c) How many cubic feet of water used would lead to a bill of \$100?

Problems 45–48 ask you to plot graphs based on the following story: "As I drove down the highway this morning, at first traffic was fast and uncongested, then it crept nearly bumper-to-bumper until we passed an accident, after which traffic flow went back to normal until I exited."

45. Driving speed against time on the highway

46. Distance driven against time on the highway

47. Distance from my exit vs time on the highway

48. Distance between cars vs distance driven on the highway

49. Let $f(t)$ be the number of US billionaires in the US in year t.

(a) Express the following statements[4] in terms of f.
(i) In 1985 there were 13 US billionaires.
(ii) In 1990 there were 99 US billionaires.
(b) Find the average yearly increase in the number of US billionaires between 1985 and 1990. Express this using f.
(c) Assuming the yearly increase remains constant, find a formula predicting the number of US billionaires in year t.

[4] //hypertextbook.com/facts/2005/MichelleLee.shtml

50. An alternative to petroleum-based diesel fuel, biodiesel, is derived from renewable resources such as food crops, algae, and animal oils. The table shows the recent annual percent growth in US biodiesel consumption.[5]

Year	2005	2006	2007	2008	2009
% growth over previous yr	237	186.6	37.2	−11.7	7.3

(a) Find the largest time interval over which the percentage growth in the US consumption of biodiesel was an increasing function of time. Interpret what increasing means, practically speaking, in this case.

(b) Find the largest time interval over which the actual US consumption of biodiesel was an increasing function of time. Interpret what increasing means, practically speaking, in this case.

51. Hydroelectric power is electric power generated by the force of moving water. Figure 1.15 shows[6] the annual percent growth in hydroelectric power consumption by the US industrial sector between 2004 and 2009.

(a) Find the largest time interval over which the percentage growth in the US consumption of hydroelectric power was a decreasing function of time. Interpret what decreasing means, practically speaking, in this case.

(b) Find the largest time interval over which the actual US consumption of hydroelectric power was a decreasing function of time. Interpret what decreasing means, practically speaking, in this case.

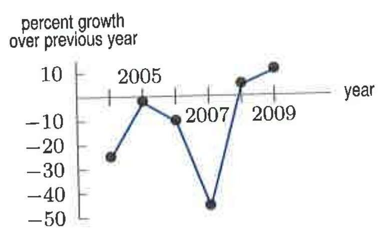


Figure 1.15

52. Solar panels are arrays of photovoltaic cells that convert solar radiation into electricity. The table shows the annual percent change in the US price per watt of a solar panel.[7]

Year	2004	2005	2006	2007	2008
% growth over previous yr	−5.7	6.7	9.7	−3.7	3.6

(a) Find the largest time interval over which the percentage growth in the US price per watt of a solar panel was an increasing function of time. Interpret what increasing means, practically speaking, in this case.

(b) Find the largest time interval over which the actual price per watt of a solar panel was an increasing function of time. Interpret what increasing means, practically speaking, in this case.

53. Table 1.4 shows the average annual sea level, S, in meters, in Aberdeen, Scotland,[8] as a function of time, t, measured in years before 2008.

Table 1.4

t	0	25	50	75	100	125
S	7.094	7.019	6.992	6.965	6.938	6.957

(a) What was the average sea level in Aberdeen in 2008?

(b) In what year was the average sea level 7.019 meters? 6.957 meters?

(c) Table 1.5 gives the average sea level, S, in Aberdeen as a function of the year, x. Complete the missing values.

Table 1.5

x	1883	?	1933	1958	1983	2008
S	?	6.938	?	6.992	?	?

54. A controversial 1992 Danish study[9] reported that men's average sperm count has decreased from 113 million per milliliter in 1940 to 66 million per milliliter in 1990.

(a) Express the average sperm count, S, as a linear function of the number of years, t, since 1940.

(b) A man's fertility is affected if his sperm count drops below about 20 million per milliliter. If the linear model found in part (a) is accurate, in what year will the average male sperm count fall below this level?

55. The table gives the average weight, w, in pounds, of American men in their sixties for height, h, in inches.[10]

(a) How do you know that the data in this table could represent a linear function?

(b) Find weight, w, as a linear function of height, h. What is the slope of the line? What are the units for the slope?

[5] http://www.eia.doe.gov/aer/renew.html. Accessed February 2011.

[6] Yearly values have been joined with segments to highlight trends in the data, however values in between years should not be inferred from the segments. From http://www.eia.doe.gov/aer/renew.html. Accessed February 2011.

[7] We use the official price per peak watt, which uses the maximum number of watts a solar panel can produce under ideal conditions. From http://www.eia.doe.gov/aer/renew.html. Accessed February 2011.

[8] www.decc.gov.uk, accessed June 2011.

[9] "Investigating the Next Silent Spring," *US News and World Report*, pp. 50–52 (March 11, 1996).

[10] Adapted from "Average Weight of Americans by Height and Age," *The World Almanac* (New Jersey: Funk and Wagnalls, 1992), p. 956.

(c) Find height, h, as a linear function of weight, w. What is the slope of the line? What are the units for the slope?

h (inches)	68	69	70	71	72	73	74	75
w (pounds)	166	171	176	181	186	191	196	201

56. An airplane uses a fixed amount of fuel for takeoff, a (different) fixed amount for landing, and a third fixed amount per mile when it is in the air. How does the total quantity of fuel required depend on the length of the trip? Write a formula for the function involved. Explain the meaning of the constants in your formula.

57. The cost of planting seed is usually a function of the number of acres sown. The cost of the equipment is a *fixed cost* because it must be paid regardless of the number of acres planted. The costs of supplies and labor vary with the number of acres planted and are called *variable costs.* Suppose the fixed costs are \$10,000 and the variable costs are \$200 per acre. Let C be the total cost, measured in thousands of dollars, and let x be the number of acres planted.

(a) Find a formula for C as a function of x.
(b) Graph C against x.
(c) Which feature of the graph represents the fixed costs? Which represents the variable costs?

58. You drive at a constant speed from Chicago to Detroit, a distance of 275 miles. About 120 miles from Chicago you pass through Kalamazoo, Michigan. Sketch a graph of your distance from Kalamazoo as a function of time.

59. (a) Consider the functions graphed in Figure 1.16(a). Find the coordinates of C.

(b) Consider the functions in Figure 1.16(b). Find the coordinates of C in terms of b.

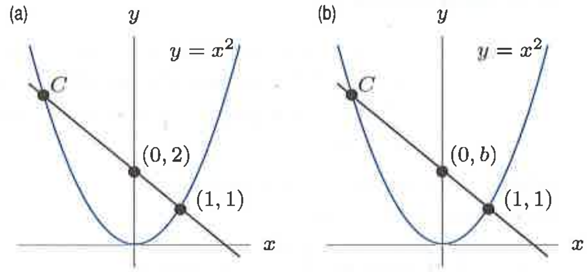


Figure 1.16

60. When Galileo was formulating the laws of motion, he considered the motion of a body starting from rest and falling under gravity. He originally thought that the velocity of such a falling body was proportional to the distance it had fallen. What do the experimental data in Table 1.6 tell you about Galileo's hypothesis? What alternative hypothesis is suggested by the two sets of data in Table 1.6 and Table 1.7?

Table 1.6

Distance (ft)	0	1	2	3	4
Velocity (ft/sec)	0	8	11.3	13.9	16

Table 1.7

Time (sec)	0	1	2	3	4
Velocity (ft/sec)	0	32	64	96	128

Strengthen Your Understanding

In Problems 61–62, explain what is wrong with the statement.

61. Values of y on the graph of $y = 0.5x - 3$ increase more slowly than values of y on the graph of $y = 0.5 - 3x$.

62. The equation $y = 2x + 1$ indicates that y is directly proportional to x with a constant of proportionality 2.

In Problems 63–64, give an example of:

63. A linear function with a positive slope and a negative x-intercept.

64. A formula representing the statement "q is inversely proportional to the cube root of p and has a positive constant of proportionality."

Are the statements in Problems 65–68 true or false? Give an explanation for your answer.

65. For any two points in the plane, there is a linear function whose graph passes through them.

66. If $y = f(x)$ is a linear function, then increasing x by 1 unit changes the corresponding y by m units, where m is the slope.

67. If y is a linear function of x, then the ratio y/x is constant for all points on the graph at which $x \neq 0$.

68. If $y = f(x)$ is a linear function, then increasing x by 2 units adds $m + 2$ units to the corresponding y, where m is the slope.

69. Which of the following functions has its domain identical with its range?

(a) $f(x) = x^2$ (b) $g(x) = \sqrt{x}$
(c) $h(x) = x^3$ (d) $i(x) = |x|$

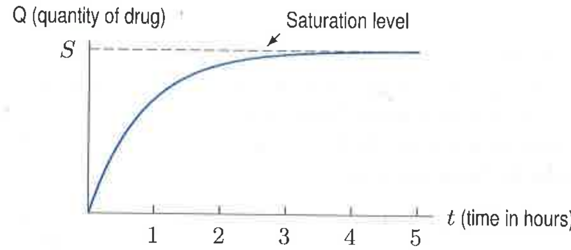


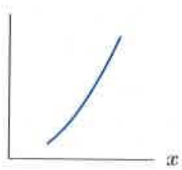
Figure 1.25: Buildup of the quantity of a drug in body

Exercises and Problems for Section 1.2

Exercises

In Exercises 1–4, decide whether the graph is concave up, concave down, or neither.

1.



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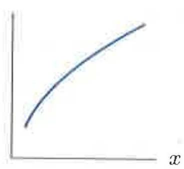

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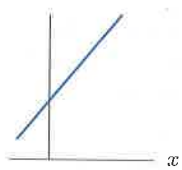

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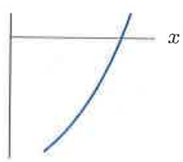


The functions in Exercises 5–8 represent exponential growth or decay. What is the initial quantity? What is the growth rate? State if the growth rate is continuous.

5. $P = 5(1.07)^t$ **6.** $P = 7.7(0.92)^t$

7. $P = 3.2e^{0.03t}$ **8.** $P = 15e^{-0.06t}$

Write the functions in Exercises 9–12 in the form $P = P_0 a^t$. Which represent exponential growth and which represent exponential decay?

9. $P = 15e^{0.25t}$ **10.** $P = 2e^{-0.5t}$

11. $P = P_0 e^{0.2t}$ **12.** $P = 7e^{-\pi t}$

In Exercises 13–14, let $f(t) = Q_0 a^t = Q_0(1+r)^t$.

(a) Find the base, a.

(b) Find the percentage growth rate, r.

13. $f(5) = 75.94$ and $f(7) = 170.86$

14. $f(0.02) = 25.02$ and $f(0.05) = 25.06$

15. A town has a population of 1000 people at time $t = 0$. In each of the following cases, write a formula for the population, P, of the town as a function of year t.

(a) The population increases by 50 people a year.

(b) The population increases by 5% a year.

16. An air-freshener starts with 30 grams and evaporates. In each of the following cases, write a formula for the quantity, Q grams, of air-freshener remaining t days after the start and sketch a graph of the function. The decrease is:

(a) 2 grams a day **(b)** 12% a day

17. For which pairs of consecutive points in Figure 1.26 is the function graphed:

(a) Increasing and concave up?

(b) Increasing and concave down?

(c) Decreasing and concave up?

(d) Decreasing and concave down?

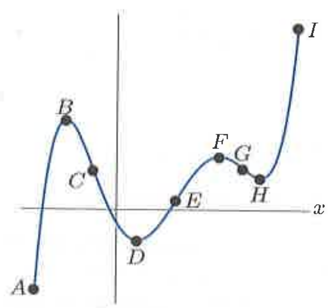


Figure 1.26

18. The table gives the average temperature in Wallingford, Connecticut, for the first 10 days in March.

(a) Over which intervals was the average temperature increasing? Decreasing?

(b) Find a pair of consecutive intervals over which the average temperature was increasing at a decreasing rate. Find another pair of consecutive intervals over which the average temperature was increasing at an increasing rate.

Day	1	2	3	4	5	6	7	8	9	10
°F	42°	42°	34°	25°	22°	34°	38°	40°	49°	49°

Problems

19. **(a)** Which (if any) of the functions in the following table could be linear? Find formulas for those functions.
(b) Which (if any) of these functions could be exponential? Find formulas for those functions.

x	$f(x)$	$g(x)$	$h(x)$
-2	12	16	37
-1	17	24	34
0	20	36	31
1	21	54	28
2	18	81	25

In Problems 20–21, find all the tables that have the given characteristic.

(A)

x	0	40	80	160
y	2.2	2.2	2.2	2.2

(B)

x	-8	-4	0	8
y	51	62	73	95

(C)

x	-4	-3	4	6
y	18	0	4.5	-2.25

(D)

x	3	4	5	6
y	18	9	4.5	2.25

20. y could be a linear function of x.

21. y could be an exponential function of x.

22. In 2010, the world's population reached 6.91 billion and was increasing at a rate of 1.1% per year. Assume that this growth rate remains constant. (In fact, the growth rate has decreased since 1987.)

(a) Write a formula for the world population (in billions) as a function of the number of years since 2010.
(b) Estimate the population of the world in the year 2020.
(c) Sketch world population as a function of years since 2010. Use the graph to estimate the doubling time of the population of the world.

23. **(a)** A population, P, grows at a continuous rate of 2% a year and starts at 1 million. Write P in the form $P = P_0 e^{kt}$, with P_0, k constants.
(b) Plot the population in part (a) against time.

24. A certain region has a population of 10,000,000 and an annual growth rate of 2%. Estimate the doubling time by guessing and checking.

25. A photocopy machine can reduce copies to 80% of their original size. By copying an already reduced copy, further reductions can be made.

(a) If a page is reduced to 80%, what percent enlargement is needed to return it to its original size?
(b) Estimate the number of times in succession that a page must be copied to make the final copy less than 15% of the size of the original.

26. When a new product is advertised, more and more people try it. However, the rate at which new people try it slows as time goes on.

(a) Graph the total number of people who have tried such a product against time.
(b) What do you know about the concavity of the graph?

27. Sketch reasonable graphs for the following. Pay particular attention to the concavity of the graphs.

(a) The total revenue generated by a car rental business, plotted against the amount spent on advertising.
(b) The temperature of a cup of hot coffee standing in a room, plotted as a function of time.

28. Each of the functions g, h, k in Table 1.10 is increasing, but each increases in a different way. Which of the graphs in Figure 1.27 best fits each function?

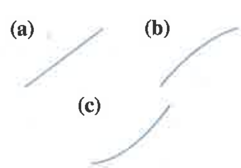


Figure 1.27

Table 1.10

t	$g(t)$	$h(t)$	$k(t)$
1	23	10	2.2
2	24	20	2.5
3	26	29	2.8
4	29	37	3.1
5	33	44	3.4
6	38	50	3.7

29. Each of the functions in Table 1.11 decreases, but each decreases in a different way. Which of the graphs in Figure 1.28 best fits each function?

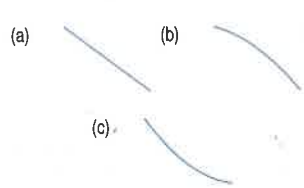


Figure 1.28

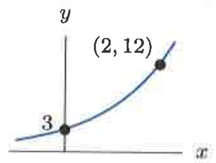
Table 1.11

x	$f(x)$	$g(x)$	$h(x)$
1	100	22.0	9.3
2	90	21.4	9.1
3	81	20.8	8.8
4	73	20.2	8.4
5	66	19.6	7.9
6	60	19.0	7.3

30. One of the main contaminants of a nuclear accident, such as that at Chernobyl, is strontium-90, which decays exponentially at a continuous rate of approximately 2.47% per year. After the Chernobyl disaster, it was suggested that it would be about 100 years before the region would again be safe for human habitation. What percent of the original strontium-90 would still remain then?

Give a possible formula for the functions in Problems 31–34.

31.



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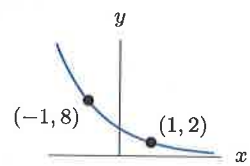

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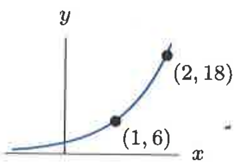
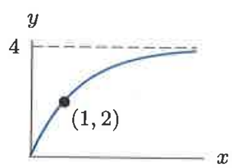


34.



35. Table 1.12 shows some values of a linear function f and an exponential function g. Find exact values (not decimal approximations) for each of the missing entries.

Table 1.12

x	0	1	2	3	4
$f(x)$	10	?	20	?	?
$g(x)$	10	?	20	?	?

36. Match the functions $h(s)$, $f(s)$, and $g(s)$, whose values are in Table 1.13, with the formulas

$$y = a(1.1)^s, \quad y = b(1.05)^s, \quad y = c(1.03)^s,$$

assuming a, b, and c are constants. Note that the function values have been rounded to two decimal places.

Table 1.13

s	$h(s)$	s	$f(s)$	s	$g(s)$
2	1.06	1	2.20	3	3.47
3	1.09	2	2.42	4	3.65
4	1.13	3	2.66	5	3.83
5	1.16	4	2.93	6	4.02
6	1.19	5	3.22	7	4.22

37. (a) Estimate graphically the doubling time of the exponentially growing population shown in Figure 1.29. Check that the doubling time is independent of where you start on the graph.
(b) Show algebraically that if $P = P_0a^t$ doubles between time t and time $t + d$, then d is the same number for any t.

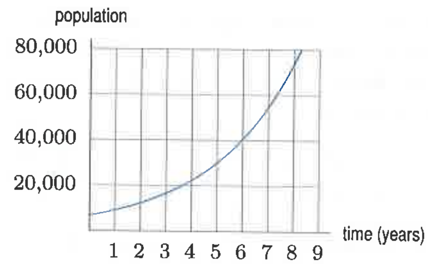


Figure 1.29

38. A deposit of P_0 into a bank account has a doubling time of 50 years. No other deposits or withdrawals are made.
(a) How much money is in the bank account after 50 years? 100 years? 150 years? (Your answer will involve P_0.)
(b) How many times does the amount of money double in t years? Use this to write a formula for P, the amount of money in the account after t years.

39. A 325 mg aspirin has a half-life of H hours in a patient's body.
(a) How long does it take for the quantity of aspirin in the patient's body to be reduced to 162.5 mg? To 81.25 mg? To 40.625 mg? (Note that $162.5 = 325/2$, etc. Your answers will involve H.)
(b) How many times does the quantity of aspirin, A mg, in the body halve in t hours? Use this to give a formula for A after t hours.

40. (a) The half-life of radium-226 is 1620 years. If the initial quantity of radium is Q_0, explain why the quantity, Q, of radium left after t years, is given by

$$Q = Q_0\left(\frac{1}{2}\right)^{t/1620}.$$

(b) What percentage of the original amount of radium is left after 500 years?

41. In the early 1960s, radioactive strontium-90 was released during atmospheric testing of nuclear weapons and got into the bones of people alive at the time. If the half-life of strontium-90 is 29 years, what fraction of the strontium-90 absorbed in 1960 remained in people's bones in 2010? [Hint: Write the function in the form $Q = Q_0(1/2)^{t/29}$.]

42. Aircraft require longer takeoff distances, called takeoff rolls, at high altitude airports because of diminished air density. The table shows how the takeoff roll for a certain light airplane depends on the airport elevation. (Takeoff rolls are also strongly influenced by air temperature; the data shown assume a temperature of 0° C.) Determine a formula for this particular aircraft that gives the takeoff roll as an exponential function of airport elevation.

Elevation (ft)	Sea level	1000	2000	3000	4000
Takeoff roll (ft)	670	734	805	882	967

Problems 43–44 concern biodiesel, a fuel derived from renewable resources such as food crops, algae, and animal oils. The table shows the percent growth over the previous year in US biodiesel consumption.[14]

Year	2003	2004	2005	2006	2007	2008	2009
% growth	−12.5	92.9	237	186.6	37.2	−11.7	7.3

43. **(a)** According to the US Department of Energy, the US consumed 91 million gallons of biodiesel in 2005. Approximately how much biodiesel (in millions of gallons) did the US consume in 2006? In 2007?

(b) Graph the points showing the annual US consumption of biodiesel, in millions of gallons of biodiesel, for the years 2005 to 2009. Label the scales on the horizontal and vertical axes.

44. **(a)** True or false: The annual US consumption of biodiesel grew exponentially from 2003 to 2005. Justify your answer without doing any calculations.

(b) According to this data, during what single year(s), if any, did the US consumption of biodiesel at least double?

(c) According to this data, during what single year(s), if any, did the US consumption of biodiesel at least triple?

45. Hydroelectric power is electric power generated by the force of moving water. The table shows the annual percent change in hydroelectric power consumption by the US industrial sector.[15]

Year	2005	2006	2007	2008	2009
% growth over previous yr	−1.9	−10	−45.4	5.1	11

(a) According to the US Department of Energy, the US industrial sector consumed about 29 trillion BTUs of hydroelectric power in 2006. Approximately how much hydroelectric power (in trillion BTUs) did the US consume in 2007? In 2005?

(b) Graph the points showing the annual US consumption of hydroelectric power, in trillion BTUs, for the years 2004 to 2009. Label the scales on the horizontal and vertical axes.

(c) According to this data, when did the largest yearly decrease, in trillion BTUs, in the US consumption of hydroelectric power occur? What was this decrease?

Problems 46–47 concern wind power, which has been used for centuries to propel ships and mill grain. Modern wind power is obtained from windmills which convert wind energy into electricity. Figure 1.30 shows the annual percent growth in US wind power consumption[16] between 2005 and 2009.

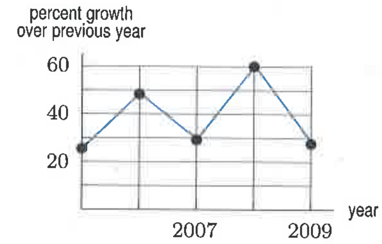


Figure 1.30

46. **(a)** According to the US Department of Energy, the US consumption of wind power was 341 trillion BTUs in 2007. How much wind power did the US consume in 2006? In 2008?

(b) Graph the points showing the annual US consumption of wind power, in trillion BTUs, for the years 2005 to 2009. Label the scales on the horizontal and vertical axes.

(c) Based on this data, in what year did the largest yearly increase, in trillion BTUs, in the US consumption of wind power occur? What was this increase?

47. **(a)** According to Figure 1.30, during what single year(s), if any, did the US consumption of wind power energy increase by at least 40%? Decrease by at least 40%?

(b) Did the US consumption of wind power energy double from 2006 to 2008?

Strengthen Your Understanding

In Problems 48–49, explain what is wrong with the statement.

48. The function $y = e^{-0.25x}$ is decreasing and its graph is concave down.

49. The function $y = 2x$ is increasing, and its graph is concave up.

In Problems 50–52, give an example of:

50. A formula representing the statement "q decreases at a constant percent rate, and $q = 2.2$ when $t = 0$."

51. A function that is increasing at a constant percent rate and that has the same vertical intercept as $f(x) = 0.3x + 2$.

52. A function with a horizontal asymptote at $y = -5$ and range $y > -5$.

[14] http://www.eia.doe.gov/aer/renew.html. Accessed February 2011.
[15] From http://www.eia.doe.gov/aer/renew.html. Accessed February 2011.
[16] Yearly values have been joined with segments to highlight trends in the data. Actual values in between years should not be inferred from the segments. From http://www.eia.doe.gov/aer/renew.html. Accessed February 2011.

Are the statements in Problems 53–59 true or false? Give an explanation for your answer.

53. The function $y = 2 + 3e^{-t}$ has a y-intercept of $y = 3$.

54. The function $y = 5 - 3e^{-4t}$ has a horizontal asymptote of $y = 5$.

55. If $y = f(x)$ is an exponential function and if increasing x by 1 increases y by a factor of 5, then increasing x by 2 increases y by a factor of 10.

56. If $y = Ab^x$ and increasing x by 1 increases y by a factor of 3, then increasing x by 2 increases y by a factor of 9.

57. An exponential function can be decreasing.

58. If a and b are positive constants, $b \neq 1$, then $y = a + ab^x$ has a horizontal asymptote.

59. The function $y = 20/(1 + 2e^{-kt})$ with $k > 0$, has a horizontal asymptote at $y = 20$.

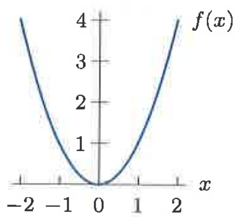
Exercises and Problems for Section 1.3

Exercises

For the functions f in Exercises 1–3, graph:

(a) $f(x+2)$ **(b)** $f(x-1)$ **(c)** $f(x)-4$
(d) $f(x+1)+3$ **(e)** $3f(x)$ **(f)** $-f(x)+1$

1.



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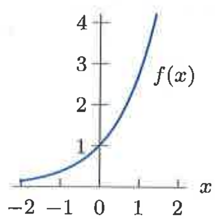
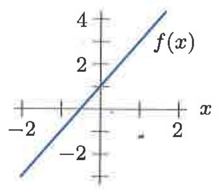


3.



In Exercises 4–7, use Figure 1.37 to graph the functions.

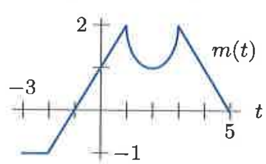


Figure 1.37

4. $n(t)=m(t)+2$

5. $p(t)=m(t-1)$

6. $k(t)=m(t+1.5)$

7. $w(t)=m(t-0.5)-2.5$

For the functions f and g in Exercises 8–11, find

(a) $f(g(1))$ **(b)** $g(f(1))$ **(c)** $f(g(x))$
(d) $g(f(x))$ **(e)** $f(t)g(t)$

8. $f(x)=x^2$, $g(x)=x+1$

9. $f(x)=\sqrt{x+4}$, $g(x)=x^2$

10. $f(x)=e^x$, $g(x)=x^2$

11. $f(x)=1/x$, $g(x)=3x+4$

12. For $g(x)=x^2+2x+3$, find and simplify:

(a) $g(2+h)$ **(b)** $g(2)$
(c) $g(2+h)-g(2)$

13. If $f(x)=x^2+1$, find and simplify:

(a) $f(t+1)$ **(b)** $f(t^2+1)$ **(c)** $f(2)$
(d) $2f(t)$ **(e)** $(f(t))^2+1$

Simplify the quantities in Exercises 14–17 using $m(z)=z^2$.

14. $m(z+1)-m(z)$

15. $m(z+h)-m(z)$

16. $m(z)-m(z-h)$

17. $m(z+h)-m(z-h)$

18. Let p be the price of an item and q be the number of items sold at that price, where $q=f(p)$. What do the following quantities mean in terms of prices and quantities sold?

(a) $f(25)$ **(b)** $f^{-1}(30)$

19. Let $C=f(A)$ be the cost, in dollars, of building a store of area A square feet. In terms of cost and square feet, what do the following quantities represent?

(a) $f(10{,}000)$ **(b)** $f^{-1}(20{,}000)$

20. Let $f(x)$ be the temperature (°F) when the column of mercury in a particular thermometer is x inches long. What is the meaning of $f^{-1}(75)$ in practical terms?

21. **(a)** Write an equation for a graph obtained by vertically stretching the graph of $y=x^2$ by a factor of 2, followed by a vertical upward shift of 1 unit. Sketch it.
(b) What is the equation if the order of the transformations (stretching and shifting) in part (a) is interchanged?
(c) Are the two graphs the same? Explain the effect of reversing the order of transformations.

22. Use Figure 1.38 to graph each of the following. Label any intercepts or asymptotes that can be determined.

(a) $y=f(x)+3$ **(b)** $y=2f(x)$
(c) $y=f(x+4)$ **(d)** $y=4-f(x)$

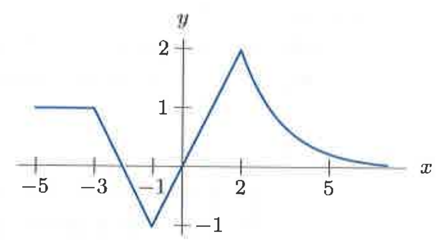
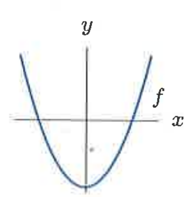


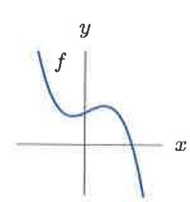
Figure 1.38

For Exercises 23–24, decide if the function $y=f(x)$ is invertible.

23.



24.



For Exercises 25–27, use a graph of the function to decide whether or not it is invertible.

25. $f(x) = x^2 + 3x + 2$

26. $f(x) = x^3 - 5x + 10$

27. $f(x) = x^3 + 5x + 10$

Are the functions in Exercises 28–35 even, odd, or neither?

28. $f(x) = x^6 + x^3 + 1$

29. $f(x) = x^3 + x^2 + x$

30. $f(x) = x^4 - x^2 + 3$

31. $f(x) = x^3 + 1$

32. $f(x) = 2x$

33. $f(x) = e^{x^2-1}$

34. $f(x) = x(x^2 - 1)$

35. $f(x) = e^x - x$

Problems

For Problems 36–39, determine functions f and g such that $h(x) = f(g(x))$. [Note: There is more than one correct answer. Do not choose $f(x) = x$ or $g(x) = x$.]

36. $h(x) = (x+1)^3$

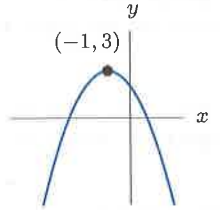
37. $h(x) = x^3 + 1$

38. $h(x) = \sqrt{x^2 + 4}$

39. $h(x) = e^{2x}$

Find possible formulas for the graphs in Problems 40–41 using shifts of x^2 or x^3.

40.



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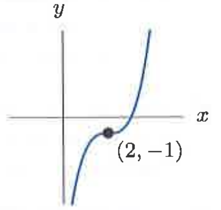


42. **(a)** Use Figure 1.39 to estimate $f^{-1}(2)$.
(b) Sketch a graph of f^{-1} on the same axes.

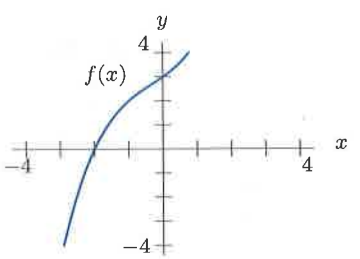


Figure 1.39

43. Write a table of values for f^{-1}, where f is as given below. The domain of f is the integers from 1 to 7. State the domain of f^{-1}.

x	1	2	3	4	5	6	7
$f(x)$	3	−7	19	4	178	2	1

For Problems 44–47, decide if the function f is invertible.

44. $f(d)$ is the total number of gallons of fuel an airplane has used by the end of d minutes of a particular flight.

45. $f(t)$ is the number of customers in Macy's department store at t minutes past noon on December 18, 2008.

46. $f(n)$ is the number of students in your calculus class whose birthday is on the n^{th} day of the year.

47. $f(w)$ is the cost of mailing a letter weighing w grams.

In Problems 48–51 the functions $r = f(t)$ and $V = g(r)$ give the radius and the volume of a commercial hot air balloon being inflated for testing. The variable t is in minutes, r is in feet, and V is in cubic feet. The inflation begins at $t = 0$. In each case, give a mathematical expression that represents the given statement.

48. The volume of the balloon t minutes after inflation began.

49. The volume of the balloon if its radius were twice as big.

50. The time that has elapsed when the radius of the balloon is 30 feet.

51. The time that has elapsed when the volume of the balloon is 10,000 cubic feet.

In Problems 52–55, use Figure 1.40 to estimate the function value or explain why it cannot be done.

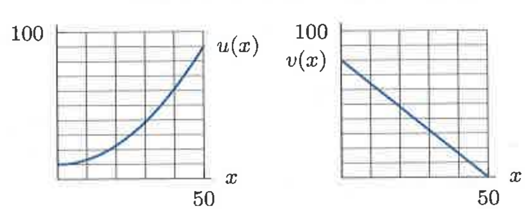


Figure 1.40

52. $u(v(10))$

53. $u(v(40))$

54. $v(u(10))$

55. $v(u(40))$

56. Figure 1.41 shows $f(t)$, the number (in millions) of motor vehicles registered[19] in the world in the year t.

(a) Is f invertible? Explain.
(b) What is the meaning of $f^{-1}(400)$ in practical terms? Evaluate $f^{-1}(400)$.
(c) Sketch the graph of f^{-1}.

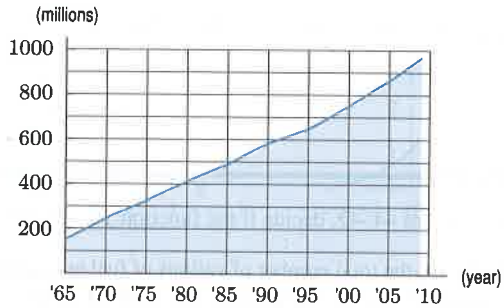


Figure 1.41

For Problems 57–62, use the graphs in Figure 1.42.

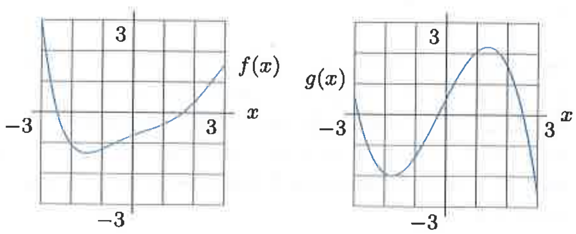


Figure 1.42

57. Estimate $f(g(1))$.

58. Estimate $g(f(2))$.

59. Estimate $f(f(1))$.

60. Graph $f(g(x))$.

61. Graph $g(f(x))$.

62. Graph $f(f(x))$.

63. Figure 1.43 is a graph of the function $f(t)$. Here $f(t)$ is the depth in meters below the Atlantic Ocean floor where t million-year-old rock can be found.[20]

(a) Evaluate $f(15)$, and say what it means in practical terms.
(b) Is f invertible? Explain.
(c) Evaluate $f^{-1}(120)$, and say what it means in practical terms.
(d) Sketch a graph of f^{-1}.

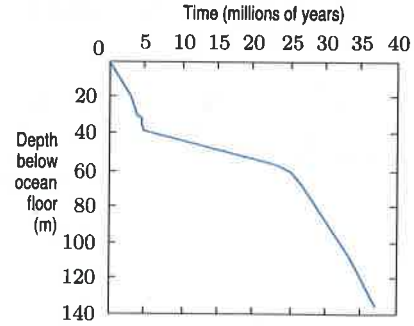


Figure 1.43

64. A tree of height y meters has, on average, B branches, where $B = y - 1$. Each branch has, on average, n leaves, where $n = 2B^2 - B$. Find the average number of leaves of a tree as a function of height.

65. A spherical balloon is growing with radius $r = 3t + 1$, in centimeters, for time t in seconds. Find the volume of the balloon at 3 seconds.

66. The cost of producing q articles is given by the function $C = f(q) = 100 + 2q$.

(a) Find a formula for the inverse function.
(b) Explain in practical terms what the inverse function tells you.

67. How does the graph of $Q = S(1 - e^{-kt})$ in Example 4 on page 16 relate to the graph of the exponential decay function, $y = Se^{-kt}$?

68. Complete the following table with values for the functions f, g, and h, given that:

(a) f is an even function.
(b) g is an odd function.
(c) h is the composition $h(x) = g(f(x))$.

x	$f(x)$	$g(x)$	$h(x)$
−3	0	0	
−2	2	2	
−1	2	2	
0	0	0	
1			
2			
3			

[19]www.earth-policy.org, accessed June 5, 2011. In 2000, about 30% of the registered vehicles were in the US.
[20]Data of Dr. Murlene Clark based on core samples drilled by the research ship *Glomar Challenger*, taken from *Initial Reports of the Deep Sea Drilling Project*.

Strengthen Your Understanding

In Problems 69–71, explain what is wrong with the statement.

69. The graph of $f(x) = -(x+1)^3$ is the graph of $g(x) = -x^3$ shifted right by 1 unit.

70. $f(x) = 3x+5$ and $g(x) = -3x-5$ are inverse functions of each other.

71. The inverse of $f(x) = x$ is $f^{-1}(x) = 1/x$.

In Problems 72–75, give an example of:

72. An invertible function whose graph contains the point $(0,3)$.

73. An even function whose graph does not contain the point $(0,0)$.

74. An increasing function $f(x)$ whose values are greater than those of its inverse function $f^{-1}(x)$ for $x > 0$.

75. Two functions $f(x)$ and $g(x)$ such that moving the graph of f to the left 2 units gives the graph of g and moving the graph of f up 3 also gives the graph of g.

Are the statements in Problems 76–83 true or false? Give an explanation for your answer.

76. The graph of $f(x) = 100(10^x)$ is a horizontal shift of the graph of $g(x) = 10^x$.

77. If f is an increasing function, then f^{-1} is an increasing function.

78. If a function is even, then it does not have an inverse.

79. If a function is odd, then it does not have an inverse.

80. The function $f(x) = e^{-x^2}$ is decreasing for all x.

81. If $g(x)$ is an even function then $f(g(x))$ is even for every function $f(x)$.

82. If $f(x)$ is an even function then $f(g(x))$ is even for every function $g(x)$.

83. There is a function which is both even and odd.

Suppose f is an increasing function and g is a decreasing function. In Problems 84–87, give an example for f and g for which the statement is true, or say why such an example is impossible.

84. $f(x) + g(x)$ is decreasing for all x.

85. $f(x) - g(x)$ is decreasing for all x.

86. $f(x)g(x)$ is decreasing for all x.

87. $f(g(x))$ is increasing for all x.

1.4 LOGARITHMIC FUNCTIONS

In Section 1.2, we approximated the population of Burkina Faso (in millions) by the function

$$P = f(t) = 12.853(1.034)^t,$$

where t is the number of years since 2003. Now suppose that instead of calculating the population at time t, we ask when the population will reach 20 million. We want to find the value of t for which

$$20 = f(t) = 12.853(1.034)^t.$$

We use logarithms to solve for a variable in an exponent.

Logarithms to Base 10 and to Base e

We define the *logarithm* function, $\log_{10} x$, to be the inverse of the exponential function, 10^x, as follows:

> The **logarithm** to base 10 of x, written $\log_{10} x$, is the power of 10 we need to get x. In other words,
>
> $$\log_{10} x = c \quad \text{means} \quad 10^c = x.$$
>
> We often write $\log x$ in place of $\log_{10} x$.

The other frequently used base is e. The logarithm to base e is called the *natural logarithm* of x, written $\ln x$ and defined to be the inverse function of e^x, as follows:

> The **natural logarithm** of x, written $\ln x$, is the power of e needed to get x. In other words,
>
> $$\ln x = c \quad \text{means} \quad e^c = x.$$

Thus,

$$f^{-1}(P) = 68.868 \log P - 76.375.$$

Note that

$$f^{-1}(20) = 68.868(\log 20) - 76.375 = 13.22,$$

which agrees with the result of Example 2.

Exercises and Problems for Section 1.4

Exercises

Simplify the expressions in Exercises 1–6 completely.

1. $e^{\ln(1/2)}$

2. $10^{\log(AB)}$

3. $5e^{\ln(A^2)}$

4. $\ln(e^{2AB})$

5. $\ln(1/e) + \ln(AB)$

6. $2\ln\left(e^A\right) + 3\ln B^e$

For Exercises 7–18, solve for x using logs.

7. $3^x = 11$

8. $17^x = 2$

9. $20 = 50(1.04)^x$

10. $4 \cdot 3^x = 7 \cdot 5^x$

11. $7 = 5e^{0.2x}$

12. $2^x = e^{x+1}$

13. $50 = 600e^{-0.4x}$

14. $2e^{3x} = 4e^{5x}$

15. $7^{x+2} = e^{17x}$

16. $10^{x+3} = 5e^{7-x}$

17. $2x - 1 = e^{\ln x^2}$

18. $4e^{2x-3} - 5 = e$

For Exercises 19–24, solve for t. Assume a and b are positive constants and k is nonzero.

19. $a = b^t$

20. $P = P_0\, a^t$

21. $Q = Q_0\, a^{nt}$

22. $P_0\, a^t = Q_0\, b^t$

23. $a = be^t$

24. $P = P_0\, e^{kt}$

In Exercises 25–28, put the functions in the form $P = P_0e^{kt}$.

25. $P = 15(1.5)^t$

26. $P = 10(1.7)^t$

27. $P = 174(0.9)^t$

28. $P = 4(0.55)^t$

Find the inverse function in Exercises 29–31.

29. $p(t) = (1.04)^t$

30. $f(t) = 50e^{0.1t}$

31. $f(t) = 1 + \ln t$

Problems

32. The population of a region is growing exponentially. There were 40,000,000 people in 2000 ($t = 0$) and 48,000,000 in 2010. Find an expression for the population at any time t, in years. What population would you predict for the year 2020? What is the doubling time?

33. One hundred kilograms of a radioactive substance decay to 40 kg in 10 years. How much remains after 20 years?

34. A culture of bacteria originally numbers 500. After 2 hours there are 1500 bacteria in the culture. Assuming exponential growth, how many are there after 6 hours?

35. The population of the US was 281.4 million in 2000 and 308.7 million in 2010.[23] Assuming exponential growth,

(a) In what year is the population expected to go over 350 million?

(b) What population is predicted for the 2020 census?

36. The concentration of the car exhaust fume nitrous oxide, NO_2, in the air near a busy road is a function of distance from the road. The concentration decays exponentially at a continuous rate of 2.54% per meter.[24] At what distance from the road is the concentration of NO_2 half what it is on the road?

37. For children and adults with diseases such as asthma, the number of respiratory deaths per year increases by 0.33% when pollution particles increase by a microgram per cubic meter of air.[25]

(a) Write a formula for the number of respiratory deaths per year as a function of quantity of pollution in the air. (Let Q_0 be the number of deaths per year with no pollution.)

(b) What quantity of air pollution results in twice as many respiratory deaths per year as there would be without pollution?

[23] http://2010.census.gov/2010census/. Accessed April 17, 2011.

[24] Rickwood, P. and Knight, D. (2009). "The health impacts of local traffic pollution on primary school age children." *State of Australian Cities 2009 Conference Proceedings*.

[25] Brook, R. D., Franklin, B., Cascio, W., Hong, Y., Howard, G., Lipsett, M., Luepker, R., Mittleman, M., Samet, J., and Smith, S. C. (2004). "Air pollution and cardiovascular disease." *Circulation*, 109(21):2655267.

38. The number of alternative fuel vehicles[26] running on E85, fuel that is up to 85% plant-derived ethanol, increased exponentially in the US between 2003 and 2008.

(a) Use this information to complete the missing table values.

(b) How many E85-powered vehicles were there in the US in 2003?

(c) By what percent did the number of E85-powered vehicles grow from 2004 to 2008?

Year	2004	2005	2006	2007	2008
Number of E85 vehicles	211,800	?	?	?	450,327

39. At time t hours after taking the cough suppressant hydrocodone bitartrate, the amount, A, in mg, remaining in the body is given by $A = 10(0.82)^t$.

(a) What was the initial amount taken?

(b) What percent of the drug leaves the body each hour?

(c) How much of the drug is left in the body 6 hours after the dose is administered?

(d) How long is it until only 1 mg of the drug remains in the body?

40. A cup of coffee contains 100 mg of caffeine, which leaves the body at a continuous rate of 17% per hour.

(a) Write a formula for the amount, A mg, of caffeine in the body t hours after drinking a cup of coffee.

(b) Graph the function from part (a). Use the graph to estimate the half-life of caffeine.

(c) Use logarithms to find the half-life of caffeine.

41. The exponential function $y(x) = Ce^{\alpha x}$ satisfies the conditions $y(0) = 2$ and $y(1) = 1$. Find the constants C and α. What is $y(2)$?

42. Without a calculator or computer, match the functions e^x, $\ln x$, x^2, and $x^{1/2}$ to their graphs in Figure 1.46.

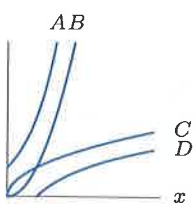


Figure 1.46

43. With time, t, in years since the start of 1980, textbook prices have increased at 6.7% per year while inflation has been 3.3% per year.[27] Assume both rates are continuous growth rates.

(a) Find a formula for $B(t)$, the price of a textbook in year t if it cost $\$B_0$ in 1980.

(b) Find a formula for $P(t)$, the price of an item in year t if it cost $\$P_0$ in 1980 and its price rose according to inflation.

(c) A textbook cost \$50 in 1980. When is its price predicted to be double the price that would have resulted from inflation alone?

44. In November 2010, a "tiger summit" was held in St. Petersburg, Russia.[28] In 1900, there were 100,000 wild tigers worldwide; in 2010 the number was 3200.

(a) Assuming the tiger population has decreased exponentially, find a formula for $f(t)$, the number of wild tigers t years since 1900.

(b) Between 2000 and 2010, the number of wild tigers decreased by 40%. Is this percentage larger or smaller than the decrease in the tiger population predicted by your answer to part (a)?

45. In 2011, the populations of China and India were approximately 1.34 and 1.19 billion people[29], respectively. However, due to central control the annual population growth rate of China was 0.4% while the population of India was growing by 1.37% each year. If these growth rates remain constant, when will the population of India exceed that of China?

46. The third-quarter revenue of Apple® went from \$3.68 billion[30] in 2005 to \$15.68 billion[31] in 2010. Find an exponential function to model the revenue as a function of years since 2005. What is the continuous percent growth rate, per year, of sales?

47. The world population was 6.9 billion at the end of 2010 and is predicted to reach 9 billion by the end of 2050.[32]

(a) Assuming the population is growing exponentially, what is the continuous growth rate per year?

(b) The United Nations celebrated the "Day of 5 Billion" on July 11, 1987, and the "Day of 6 Billion" on October 12, 1999. Using the growth rate in part (a), when is the "Day of 7 Billion" predicted to be?

[26] http://www.eia.doe.gov/aer/renew.html
[27] Data from "Textbooks headed for ash heap of history", http://educationtechnews.com, Vol 5, 2010.
[28] "Tigers would be extinct in Russia if unprotected," Yahoo! News, Nov. 21, 2010.
[29] http://www.indexmundi.com/. Accessed April 17, 2011.
[30] http://www.apple.com/pr/library/2005/oct/11results.html. Accessed April 27, 2011.
[31] http://www.apple.com/pr/library/2010/01/25results.html. Accessed April 27, 2011.
[32] "Reviewing the Bidding on the Climate Files", in About Dot Earth, *New York Times*, Nov. 19, 2010.

48. In the early 1920s, Germany had tremendously high inflation, called hyperinflation. Photographs of the time show people going to the store with wheelbarrows full of money. If a loaf of bread cost $1/4$ marks in 1919 and 2,400,000 marks in 1922, what was the average yearly inflation rate between 1919 and 1922?

49. Different isotopes (versions) of the same element can have very different half-lives. With t in years, the decay of plutonium-240 is described by the formula

$$Q = Q_0 e^{-0.00011t},$$

whereas the decay of plutonium-242 is described by

$$Q = Q_0 e^{-0.0000018t}.$$

Find the half-lives of plutonium-240 and plutonium-242.

50. The size of an exponentially growing bacteria colony doubles in 5 hours. How long will it take for the number of bacteria to triple?

51. Air pressure, P, decreases exponentially with height, h, above sea level. If P_0 is the air pressure at sea level and h is in meters, then

$$P = P_0 e^{-0.00012h}.$$

(a) At the top of Mount McKinley, height 6194 meters (about 20,320 feet), what is the air pressure, as a percent of the pressure at sea level?
(b) The maximum cruising altitude of an ordinary commercial jet is around 12,000 meters (about 39,000 feet). At that height, what is the air pressure, as a percent of the sea level value?

52. Find the equation of the line l in Figure 1.47.

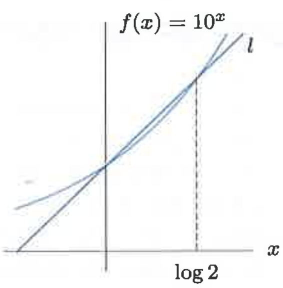


Figure 1.47

53. In 2010, there were about 246 million vehicles (cars and trucks) and about 308.7 million people in the US.[33] The number of vehicles grew 15.5% over the previous decade, while the population has been growing at 9.7% per decade. If the growth rates remain constant, when will there be, on average, one vehicle per person?

54. A picture supposedly painted by Vermeer (1632–1675) contains 99.5% of its carbon-14 (half-life 5730 years). From this information decide whether the picture is a fake. Explain your reasoning.

55. Is there a difference between $\ln[\ln(x)]$ and $\ln^2(x)$? [Note: $\ln^2(x)$ is another way of writing $(\ln x)^2$.]

56. If $h(x) = \ln(x + a)$, where $a > 0$, what is the effect of increasing a on

(a) The y-intercept? **(b)** The x-intercept?

57. If $h(x) = \ln(x + a)$, where $a > 0$, what is the effect of increasing a on the vertical asymptote?

58. If $g(x) = \ln(ax + 2)$, where $a \neq 0$, what is the effect of increasing a on

(a) The y-intercept? **(b)** The x-intercept?

59. If $f(x) = a\ln(x + 2)$, what is the effect of increasing a on the vertical asymptote?

60. If $g(x) = \ln(ax + 2)$, where $a \neq 0$, what is the effect of increasing a on the vertical asymptote?

Strengthen Your Understanding

In Problems 61–62, explain what is wrong with the statement.

61. The function $-\log|x|$ is odd.

62. For all $x > 0$, the value of $\ln(100x)$ is 100 times larger than $\ln x$.

In Problems 63–64, give an example of:

63. A function $f(x)$ such that $\ln(f(x))$ is only defined for $x < 0$.

64. A function with a vertical asymptote at $x = 3$ and defined only for $x > 3$.

Are the statements in Problems 65–68 true or false? Give an explanation for your answer.

65. The graph of $f(x) = \ln x$ is concave down.

66. The graph of $g(x) = \log(x - 1)$ crosses the x-axis at $x = 1$.

67. The inverse function of $y = \log x$ is $y = 1/\log x$.

68. If a and b are positive constants, then $y = \ln(ax + b)$ has no vertical asymptote.

[33]http://www.autoblog.com/2010/01/04/report-number-of-cars-in-the-u-s-dropped-by-four-million-in-20/ and http://2010.census.gov/news/releases/operations/cb10-cn93.html. Accessed February 2012.

Exercises and Problems for Section 1.5

Exercises

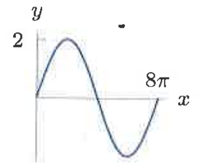
For Exercises 1–9, draw the angle using a ray through the origin, and determine whether the sine, cosine, and tangent of that angle are positive, negative, zero, or undefined.

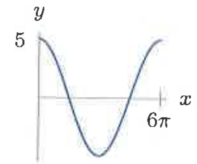
1. $\frac{3\pi}{2}$ **2.** 2π **3.** $\frac{\pi}{4}$
4. 3π **5.** $\frac{\pi}{6}$ **6.** $\frac{4\pi}{3}$
7. $\frac{-4\pi}{3}$ **8.** 4 **9.** -1

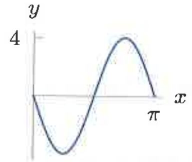
Find the period and amplitude in Exercises 10–13.

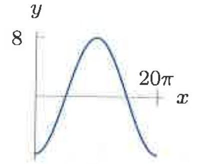
10. $y = 7\sin(3t)$ **11.** $z = 3\cos(u/4) + 5$
12. $w = 8 - 4\sin(2x + \pi)$ **13.** $r = 0.1\sin(\pi t) + 2$

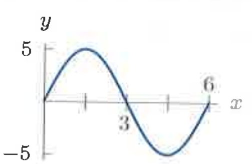
For Exercises 14–23, find a possible formula for each graph.

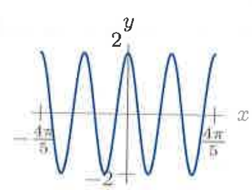
14.


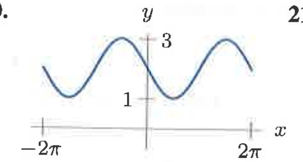
15.


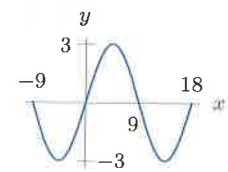
16.


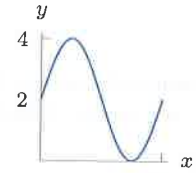
17.


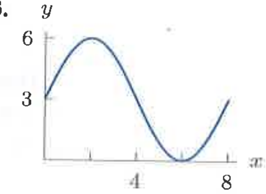
18.


19.


20.


21.


22.


23.


In Exercises 24–26, calculate the quantity without using the the trigonometric functions on your calculator. You are given that $\sin(\pi/12) = 0.259$ and $\cos(\pi/5) = 0.809$. You may want to draw a picture showing the angles involved and check your answer on a calculator.

24. $\cos\left(-\frac{\pi}{5}\right)$ **25.** $\sin\frac{\pi}{5}$ **26.** $\cos\frac{\pi}{12}$

In Exercises 27–31, find a solution to the equation if possible. Give the answer in exact form and in decimal form.

27. $2 = 5\sin(3x)$ **28.** $1 = 8\cos(2x + 1) - 3$
29. $8 = 4\tan(5x)$ **30.** $1 = 8\tan(2x + 1) - 3$
31. $8 = 4\sin(5x)$

Problems

32. Without a calculator or computer, match the formulas with the graphs in Figure 1.61.

(a) $y = 2\cos(t - \pi/2)$ **(b)** $y = 2\cos t$
(c) $y = 2\cos(t + \pi/2)$

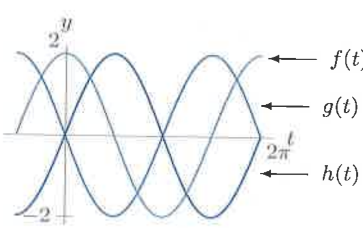


Figure 1.61

33. What is the difference between $\sin x^2$, $\sin^2 x$, and $\sin(\sin x)$? Express each of the three as a composition. (Note: $\sin^2 x$ is another way of writing $(\sin x)^2$.)

34. On the graph of $y = \sin x$, points P and Q are at consecutive lowest and highest points. Find the slope of the line through P and Q.

35. A population of animals oscillates sinusoidally between a low of 700 on January 1 and a high of 900 on July 1.

(a) Graph the population against time.
(b) Find a formula for the population as a function of time, t, in months since the start of the year.

36. The desert temperature, H, oscillates daily between 40°F at 5 am and 80°F at 5 pm. Write a possible formula for H in terms of t, measured in hours from 5 am.

37. **(a)** Match the functions $\omega = f(t), \omega = g(t), \omega = h(t), \omega = k(t)$, whose values are in the table, with the functions with formulas:

(i) $\omega = 1.5 + \sin t$ (ii) $\omega = 0.5 + \sin t$
(iii) $\omega = -0.5 + \sin t$ (iv) $\omega = -1.5 + \sin t$

(b) Based on the table, what is the relationship between the values of $g(t)$ and $k(t)$? Explain this relationship using the formulas you chose for g and k.

(c) Using the formulas you chose for g and h, explain why all the values of g are positive, whereas all the values of h are negative.

t	$f(t)$	t	$g(t)$	t	$h(t)$	t	$k(t)$
6.0	−0.78	3.0	1.64	5.0	−2.46	3.0	0.64
6.5	−0.28	3.5	1.15	5.1	−2.43	3.5	0.15
7.0	0.16	4.0	0.74	5.2	−2.38	4.0	−0.26
7.5	0.44	4.5	0.52	5.3	−2.33	4.5	−0.48
8.0	0.49	5.0	0.54	5.4	−2.27	5.0	−0.46

38. The depth of water in a tank oscillates sinusoidally once every 6 hours. If the smallest depth is 5.5 feet and the largest depth is 8.5 feet, find a possible formula for the depth in terms of time in hours.

39. The voltage, V, of an electrical outlet in a home as a function of time, t (in seconds), is $V = V_0 \cos(120\pi t)$.

(a) What is the period of the oscillation?
(b) What does V_0 represent?
(c) Sketch the graph of V against t. Label the axes.

40. The power output, P, of a solar panel varies with the position of the sun. Let $P = 10 \sin\theta$ watts, where θ is the angle between the sun's rays and the panel, $0 \le \theta \le \pi$. On a typical summer day in Ann Arbor, Michigan, the sun rises at 6 am and sets at 8 pm and the angle is $\theta = \pi t/14$, where t is time in hours since 6 am and $0 \le t \le 14$.

(a) Write a formula for a function, $f(t)$, giving the power output of the solar panel (in watts) t hours after 6 am on a typical summer day in Ann Arbor.
(b) Graph the function $f(t)$ in part (a) for $0 \le t \le 14$.
(c) At what time is the power output greatest? What is the power output at this time?
(d) On a typical winter day in Ann Arbor, the sun rises at 8 am and sets at 5 pm. Write a formula for a function, $g(t)$, giving the power output of the solar panel (in watts) t hours after 8 am on a typical winter day.

41. A baseball hit at an angle of θ to the horizontal with initial velocity v_0 has horizontal range, R, given by

$$R = \frac{v_0^2}{g}\sin(2\theta).$$

Here g is the acceleration due to gravity. Sketch R as a function of θ for $0 \le \theta \le \pi/2$. What angle gives the maximum range? What is the maximum range?

42. The visitors' guide to St. Petersburg, Florida, contains the chart shown in Figure 1.62 to advertise their good weather. Fit a trigonometric function approximately to the data, where H is temperature in degrees Fahrenheit, and the independent variable is time in months. In order to do this, you will need to estimate the amplitude and period of the data, and when the maximum occurs. (There are many possible answers to this problem, depending on how you read the graph.)

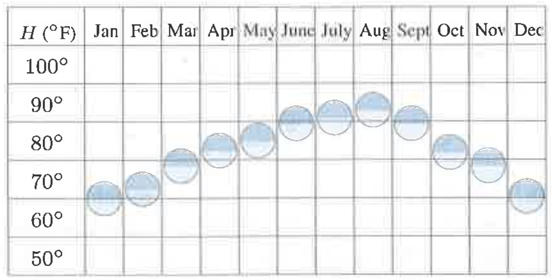


Figure 1.62: "St. Petersburg...where we're famous for our wonderful weather and year-round sunshine." (Reprinted with permission)

43. The Bay of Fundy in Canada has the largest tides in the world. The difference between low and high water levels is 15 meters (nearly 50 feet). At a particular point the depth of the water, y meters, is given as a function of time, t, in hours since midnight by

$$y = D + A\cos(B(t - C)).$$

(a) What is the physical meaning of D?
(b) What is the value of A?
(c) What is the value of B? Assume the time between successive high tides is 12.4 hours.
(d) What is the physical meaning of C?

44. A compact disc spins at a rate of 200 to 500 revolutions per minute. What are the equivalent rates measured in radians per second?

45. When a car's engine makes less than about 200 revolutions per minute, it stalls. What is the period of the rotation of the engine when it is about to stall?

46. What is the period of the earth's revolution around the sun?

47. What is the approximate period of the moon's revolution around the earth?

48. For a boat to float in a tidal bay, the water must be at least 2.5 meters deep. The depth of water around the boat, $d(t)$, in meters, where t is measured in hours since midnight, is

$$d(t) = 5 + 4.6\sin(0.5t).$$

(a) What is the period of the tides in hours?
(b) If the boat leaves the bay at midday, what is the latest time it can return before the water becomes too shallow?

49. Match graphs A-D in Figure 1.63 with the functions below. Assume a, b, c and d are positive constants.

$$f(t) = \sin t + b \qquad h(t) = \sin t + e^{ct} + d$$
$$g(t) = \sin t + at + b \qquad r(t) = \sin t - e^{ct} + b$$

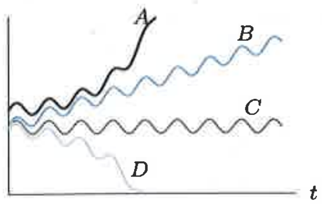


Figure 1.63

50. In Figure 1.64, the blue curve shows monthly mean carbon dioxide (CO_2) concentration, in parts per million (ppm) at Mauna Loa Observatory, Hawaii, as a function of t, in months, since December 2005. The black curve shows the monthly mean concentration adjusted for seasonal CO_2 variation.[35]

(a) Approximately how much did the monthly mean CO_2 increase between December 2005 and December 2010?

(b) Find the average monthly rate of increase of the monthly mean CO_2 between December 2005 and December 2010. Use this information to find a linear function that approximates the black curve.

(c) The seasonal CO_2 variation between December 2005 and December 2010 can be approximated by a sinusoidal function. What is the approximate period of the function? What is its amplitude? Give a formula for the function.

(d) The blue curve may be approximated by a function of the form $h(t) = f(t) + g(t)$, where $f(t)$ is sinusoidal and $g(t)$ is linear. Using your work in parts (b) and (c), find a possible formula for $h(t)$. Graph $h(t)$ using the scale in Figure 1.64.

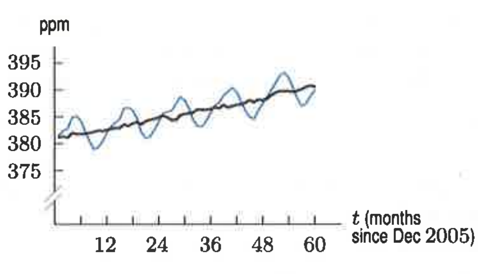


Figure 1.64

51. **(a)** Use a graphing calculator or computer to estimate the period of $2\sin\theta + 3\cos(2\theta)$.

(b) Explain your answer, given that the period of $\sin\theta$ is 2π and the period of $\cos(2\theta)$ is π.

52. Find the area of the trapezoidal cross-section of the irrigation canal shown in Figure 1.65.

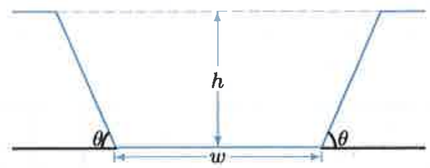


Figure 1.65

53. Graph $y = \sin x$, $y = 0.4$, and $y = -0.4$.

(a) From the graph, estimate to one decimal place all the solutions of $\sin x = 0.4$ with $-\pi \le x \le \pi$.

(b) Use a calculator to find $\arcsin(0.4)$. What is the relation between $\arcsin(0.4)$ and each of the solutions you found in part (a)?

(c) Estimate all the solutions to $\sin x = -0.4$ with $-\pi \le x \le \pi$ (again, to one decimal place).

(d) What is the relation between $\arcsin(0.4)$ and each of the solutions you found in part (c)?

54. Find the angle, in degrees, that a wheelchair ramp makes with the ground if the ramp rises 1 foot over a horizontal distance of

(a) 12 ft, the normal requirement[36]

(b) 8 ft, the steepest ramp legally permitted

(c) 20 ft, the recommendation if snow can be expected on the ramp

55. This problem introduces the arccosine function, or inverse cosine, denoted by $\boxed{\cos^{-1}}$ on most calculators.

(a) Using a calculator set in radians, make a table of values, to two decimal places, of $g(x) = \arccos x$, for $x = -1, -0.8, -0.6, \ldots, 0, \ldots, 0.6, 0.8, 1$.

(b) Sketch the graph of $g(x) = \arccos x$.

(c) Why is the domain of the arccosine the same as the domain of the arcsine?

(d) What is the range of the arccosine?

(e) Why is the range of the arccosine *not* the same as the range of the arcsine?

[35] http://www.esrl.noaa.gov/gmd/ccgg/trends/. Accessed March 2011. Monthly means joined by segments to highlight trends.

[36] http://www.access-board.gov/adaag/html/adaag.htm#4.1.6(3)a, accessed June 6, 2011.

Strengthen Your Understanding

In Problems 56–57, explain what is wrong with the statement.

56. For the function $f(x) = \sin(Bx)$ with $B > 0$, increasing the value of B increases the period.

57. For positive A, B, C, the maximum value of the function $y = A\sin(Bx) + C$ is $y = A$.

In Problems 58–59, give an example of:

58. A sine function with period 23.

59. A cosine function which oscillates between values of 1200 and 2000.

Are the statements in Problems 60–72 true or false? Give an explanation for your answer.

60. The function $f(\theta) = \cos\theta - \sin\theta$ is increasing on $0 \le \theta \le \pi/2$.

61. The function $f(t) = \sin(0.05\pi t)$ has period 0.05.

62. If t is in seconds, $g(t) = \cos(200\pi t)$ executes 100 cycles in one second.

63. The function $f(\theta) = \tan(\theta - \pi/2)$ is not defined at $\theta = \pi/2, 3\pi/2, 5\pi/2 \ldots$

64. $\sin|x| = \sin x$ for $-2\pi < x < 2\pi$

65. $\sin|x| = |\sin x|$ for $-2\pi < x < 2\pi$

66. $\cos|x| = |\cos x|$ for $-2\pi < x < 2\pi$

67. $\cos|x| = \cos x$ for $-2\pi < x < 2\pi$

68. The function $f(x) = \sin(x^2)$ is periodic, with period 2π.

69. The function $g(\theta) = e^{\sin\theta}$ is periodic.

70. If $f(x)$ is a periodic function with period k, then $f(g(x))$ is periodic with period k for every function $g(x)$.

71. If $g(x)$ is a periodic function, then $f(g(x))$ is periodic for every function $f(x)$.

72. The function $f(x) = |\sin x|$ is even.

1.6 POWERS, POLYNOMIALS, AND RATIONAL FUNCTIONS

Power Functions

A *power function* is a function in which the dependent variable is proportional to a power of the independent variable:

> A **power function** has the form
>
> $f(x) = kx^p,$ where k and p are constant.

For example, the volume, V, of a sphere of radius r is given by

$$V = g(r) = \frac{4}{3}\pi r^3.$$

As another example, the gravitational force, F, on a unit mass at a distance r from the center of the earth is given by Newton's Law of Gravitation, which says that, for some positive constant k,

$$F = \frac{k}{r^2} \quad \text{or} \quad F = kr^{-2}.$$

We consider the graphs of the power functions x^n, with n a positive integer. Figures 1.66 and 1.67 show that the graphs fall into two groups: odd and even powers. For n greater than 1, the odd powers have a "seat" at the origin and are increasing everywhere else. The even powers are first decreasing and then increasing. For large x, the higher the power of x, the faster the function climbs.

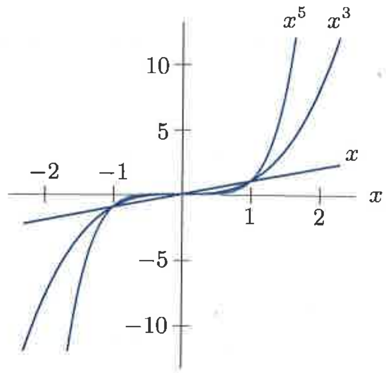


Figure 1.66: Odd powers of x: "Seat" shaped for $n > 1$

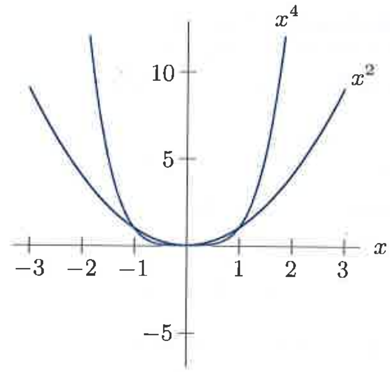


Figure 1.67: Even powers of x: $\bigcup$-shaped

Exponentials and Power Functions: Which Dominate?

In everyday language, the word exponential is often used to imply very fast growth. But do exponential functions always grow faster than power functions? To determine what happens "in the long run," we often want to know which functions *dominate* as x gets arbitrarily large.

Let's consider $y = 2^x$ and $y = x^3$. The close-up view in Figure 1.68(a) shows that between $x = 2$ and $x = 4$, the graph of $y = 2^x$ lies below the graph of $y = x^3$. The far-away view in Figure 1.68(b) shows that the exponential function $y = 2^x$ eventually overtakes $y = x^3$. Figure 1.68(c), which gives a very far-away view, shows that, for large x, the value of x^3 is insignificant compared to 2^x. Indeed, 2^x is growing so much faster than x^3 that the graph of 2^x appears almost vertical in comparison to the more leisurely climb of x^3.

We say that Figure 1.68(a) gives a *local* view of the functions' behavior, whereas Figure 1.68(c) gives a *global* view.

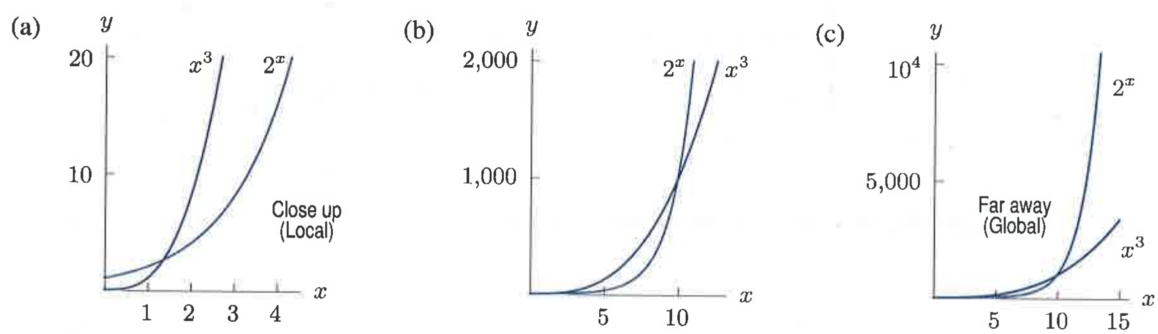


Figure 1.68: Comparison of $y = 2^x$ and $y = x^3$: Notice that $y = 2^x$ eventually dominates $y = x^3$

In fact, *every* exponential growth function eventually dominates *every* power function. Although an exponential function may be below a power function for some values of x, if we look at large enough x-values, a^x (with $a > 1$) will eventually dominate x^n, no matter what n is.

Polynomials

Polynomials are the sums of power functions with nonnegative integer exponents:

$$y = p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0.$$

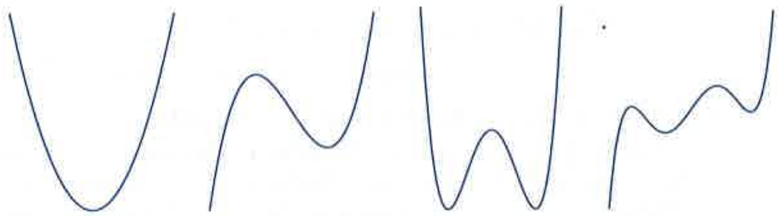


Figure 1.69: Graphs of typical polynomials of degree n

Here n is a nonnegative integer called the *degree* of the polynomial, and $a_n, a_{n-1}, \ldots, a_1, a_0$ are constants, with leading coefficient $a_n \neq 0$. An example of a polynomial of degree $n = 3$ is

$$y = p(x) = 2x^3 - x^2 - 5x - 7.$$

In this case $a_3 = 2, a_2 = -1, a_1 = -5$, and $a_0 = -7$. The shape of the graph of a polynomial depends on its degree; typical graphs are shown in Figure 1.69. These graphs correspond to a positive coefficient for x^n; a negative leading coefficient turns the graph upside down. Notice that the quadratic "turns around" once, the cubic "turns around" twice, and the quartic (fourth degree) "turns around" three times. An n^{th} degree polynomial "turns around" at most $n - 1$ times (where n is a positive integer), but there may be fewer turns.

Example 1 Find possible formulas for the polynomials whose graphs are in Figure 1.70.

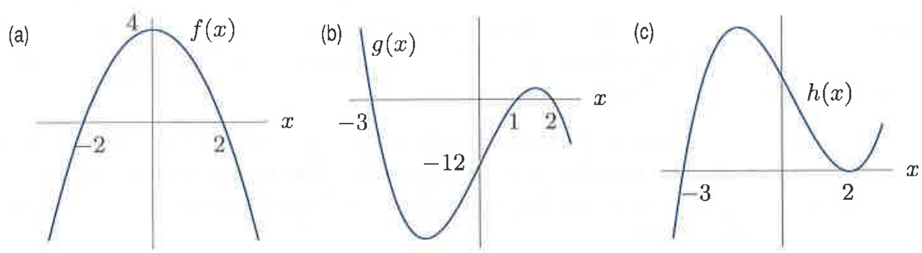


Figure 1.70: Graphs of polynomials

Solution

(a) This graph appears to be a parabola, turned upside down, and moved up by 4, so

$$f(x) = -x^2 + 4.$$

The negative sign turns the parabola upside down and the $+4$ moves it up by 4. Notice that this formula does give the correct x-intercepts since $0 = -x^2 + 4$ has solutions $x = \pm 2$. These values of x are called *zeros* of f.

We can also solve this problem by looking at the x-intercepts first, which tell us that $f(x)$ has factors of $(x + 2)$ and $(x - 2)$. So

$$f(x) = k(x + 2)(x - 2).$$

To find k, use the fact that the graph has a y-intercept of 4, so $f(0) = 4$, giving

$$4 = k(0 + 2)(0 - 2),$$

or $k = -1$. Therefore, $f(x) = -(x + 2)(x - 2)$, which multiplies out to $-x^2 + 4$.

Note that $f(x) = 4 - x^4/4$ also has the same basic shape, but is flatter near $x = 0$. There are many possible answers to these questions.

(b) This looks like a cubic with factors $(x + 3)$, $(x - 1)$, and $(x - 2)$, one for each intercept:

$$g(x) = k(x + 3)(x - 1)(x - 2).$$

Since the y-intercept is -12, we have

$$-12 = k(0 + 3)(0 - 1)(0 - 2).$$

So $k = -2$, and we get the cubic polynomial

$$g(x) = -2(x+3)(x-1)(x-2).$$

(c) This also looks like a cubic with zeros at $x = 2$ and $x = -3$. Notice that at $x = 2$ the graph of $h(x)$ touches the x-axis but does not cross it, whereas at $x = -3$ the graph crosses the x-axis. We say that $x = 2$ is a *double zero*, but that $x = -3$ is a single zero.

To find a formula for $h(x)$, imagine the graph of $h(x)$ to be slightly lower down, so that the graph has one x-intercept near $x = -3$ and two near $x = 2$, say at $x = 1.9$ and $x = 2.1$. Then a formula would be

$$h(x) \approx k(x+3)(x-1.9)(x-2.1).$$

Now move the graph back to its original position. The zeros at $x = 1.9$ and $x = 2.1$ move toward $x = 2$, giving

$$h(x) = k(x+3)(x-2)(x-2) = k(x+3)(x-2)^2.$$

The double zero leads to a repeated factor, $(x-2)^2$. Notice that when $x > 2$, the factor $(x-2)^2$ is positive, and when $x < 2$, the factor $(x-2)^2$ is still positive. This reflects the fact that $h(x)$ does not change sign near $x = 2$. Compare this with the behavior near the single zero at $x = -3$, where h does change sign.

We cannot find k, as no coordinates are given for points off of the x-axis. Any positive value of k stretches the graph vertically but does not change the zeros, so any positive k works.

Example 2 Using a calculator or computer, graph $y = x^4$ and $y = x^4 - 15x^2 - 15x$ for $-4 \le x \le 4$ and for $-20 \le x \le 20$. Set the y range to $-100 \le y \le 100$ for the first domain, and to $-100 \le y \le 200{,}000$ for the second. What do you observe?

Solution From the graphs in Figure 1.71 we see that close up ($-4 \le x \le 4$) the graphs look different; from far away, however, they are almost indistinguishable. The reason is that the leading terms (those with the highest power of x) are the same, namely x^4, and for large values of x, the leading term dominates the other terms.

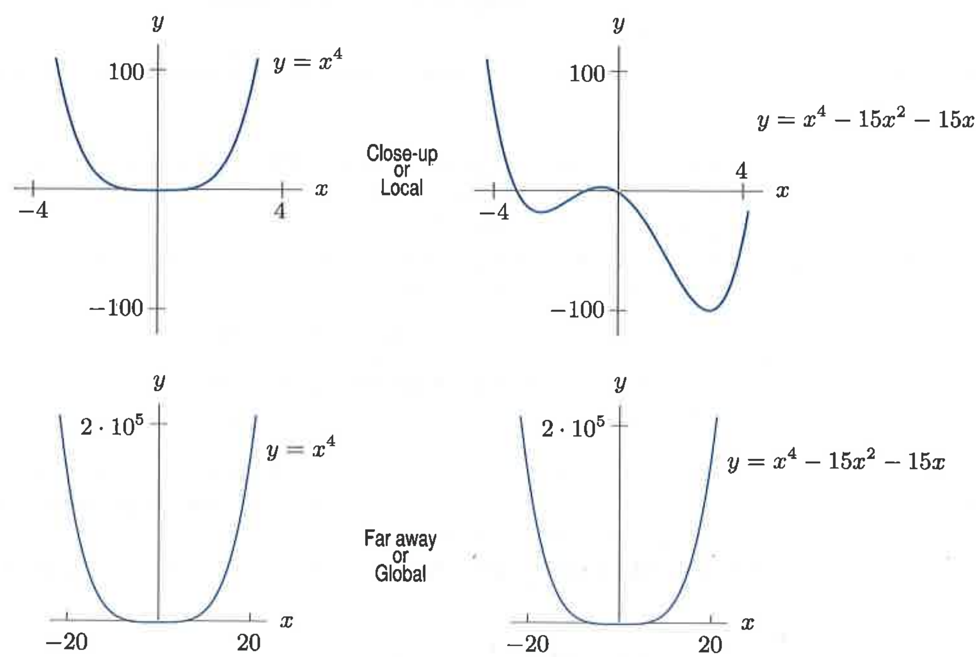


Figure 1.71: Local and global views of $y = x^4$ and $y = x^4 - 15x^2 - 15x$

Rational Functions

Rational functions are ratios of polynomials, p and q:

$$f(x) = \frac{p(x)}{q(x)}.$$

Example 3 Look at a graph and explain the behavior of $y = \dfrac{1}{x^2+4}$.

Solution The function is even, so the graph is symmetric about the y-axis. As x gets larger, the denominator gets larger, making the value of the function closer to 0. Thus the graph gets arbitrarily close to the x-axis as x increases without bound. See Figure 1.72.

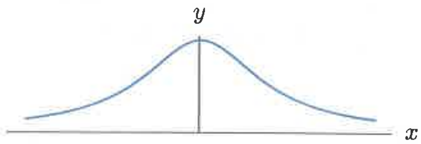


Figure 1.72: Graph of $y = \frac{1}{x^2+4}$

In the previous example, we say that $y = 0$ (i.e. the x-axis) is a *horizontal asymptote*. Writing "$\to$" to mean "tends to," we have $y \to 0$ as $x \to \infty$ and $y \to 0$ as $x \to -\infty$.

> If the graph of $y = f(x)$ approaches a horizontal line $y = L$ as $x \to \infty$ or $x \to -\infty$, then the line $y = L$ is called a **horizontal asymptote.**[37] This occurs when
>
> $$f(x) \to L \quad \text{as} \quad x \to \infty \qquad \text{or} \qquad f(x) \to L \quad \text{as} \quad x \to -\infty.$$
>
> If the graph of $y = f(x)$ approaches the vertical line $x = K$ as $x \to K$ from one side or the other, that is, if
>
> $$y \to \infty \quad \text{or} \quad y \to -\infty \quad \text{when} \quad x \to K,$$
>
> then the line $x = K$ is called a **vertical asymptote.**

The graphs of rational functions may have vertical asymptotes where the denominator is zero. For example, the function in Example 3 has no vertical asymptotes as the denominator is never zero. The function in Example 4 has two vertical asymptotes corresponding to the two zeros in the denominator.

Rational functions have horizontal asymptotes if $f(x)$ approaches a finite number as $x \to \infty$ or $x \to -\infty$. We call the behavior of a function as $x \to \pm\infty$ its *end behavior.*

Example 4 Look at a graph and explain the behavior of $y = \dfrac{3x^2-12}{x^2-1}$, including end behavior.

Solution Factoring gives

$$y = \frac{3x^2-12}{x^2-1} = \frac{3(x+2)(x-2)}{(x+1)(x-1)}$$

so $x = \pm 1$ are vertical asymptotes. If $y = 0$, then $3(x+2)(x-2) = 0$ or $x = \pm 2$; these are the x-intercepts. Note that zeros of the denominator give rise to the vertical asymptotes, whereas zeros of the numerator give rise to x-intercepts. Substituting $x = 0$ gives $y = 12$; this is the y-intercept. The function is even, so the graph is symmetric about the y-axis.

[37] We are assuming that $f(x)$ gets arbitrarily close to L as $x \to \infty$.

Table 1.18 *Values of* $y = \frac{3x^2-12}{x^2-1}$

x	$y = \frac{3x^2-12}{x^2-1}$
± 10	2.909091
± 100	2.999100
± 1000	2.999991

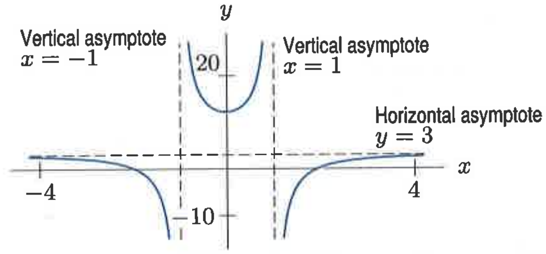


Figure 1.73: Graph of the function $y = \frac{3x^2-12}{x^2-1}$

To see what happens as $x \to \pm\infty$, look at the y-values in Table 1.18. Clearly y is getting closer to 3 as x gets large positively or negatively. Alternatively, realize that as $x \to \pm\infty$, only the highest powers of x matter. For large x, the 12 and the 1 are insignificant compared to x^2, so

$$y = \frac{3x^2 - 12}{x^2 - 1} \approx \frac{3x^2}{x^2} = 3 \quad \text{for large } x.$$

So $y \to 3$ as $x \to \pm\infty$, and therefore the horizontal asymptote is $y = 3$. See Figure 1.73. Since, for $x > 1$, the value of $(3x^2 - 12)/(x^2 - 1)$ is less than 3, the graph lies *below* its asymptote. (Why doesn't the graph lie below $y = 3$ when $-1 < x < 1$?)

Exercises and Problems for Section 1.6

Exercises

For Exercises 1–2, what happens to the value of the function as $x \to \infty$ and as $x \to -\infty$?

1. $y = 0.25x^3 + 3$ **2.** $y = 2 \cdot 10^{4x}$

In Exercises 3–10, determine the end behavior of each function as $x \to +\infty$ and as $x \to -\infty$.

3. $f(x) = -10x^4$

4. $f(x) = 3x^5$

5. $f(x) = 5x^4 - 25x^3 - 62x^2 + 5x + 300$

6. $f(x) = 1000 - 38x + 50x^2 - 5x^3$

7. $f(x) = \dfrac{3x^2 + 5x + 6}{x^2 - 4}$

8. $f(x) = \dfrac{10 + 5x^2 - 3x^3}{2x^3 - 4x + 12}$

9. $f(x) = 3x^{-4}$

10. $f(x) = e^x$

In Exercises 11–16, which function dominates as $x \to \infty$?

11. $1000x^4$ or $0.2x^5$

12. $10e^{0.1x}$ or $5000x^2$

13. $100x^5$ or 1.05^x

14. $2x^4$ or $10x^3 + 25x^2 + 50x + 100$

15. $20x^4 + 100x^2 + 5x$ or $25 - 40x^2 + x^3 + 3x^5$

16. $\sqrt{x}$ or $\ln x$

17. Each of the graphs in Figure 1.74 is of a polynomial. The windows are large enough to show end behavior.

(a) What is the minimum possible degree of the polynomial?

(b) Is the leading coefficient of the polynomial positive or negative?

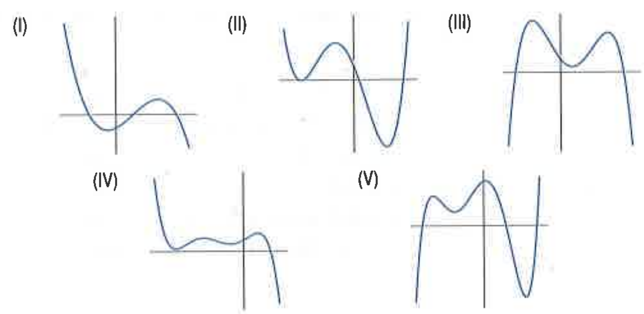
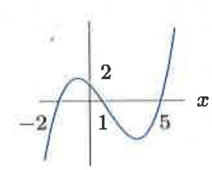


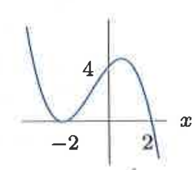
Figure 1.74

Find cubic polynomials for the graphs in Exercises 18–19.

18.

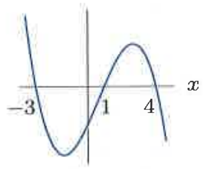


19.



Find possible formulas for the graphs in Exercises 20–23.

20.



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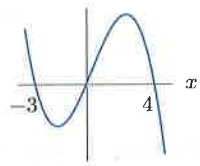

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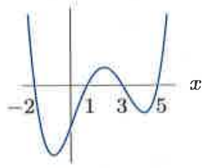
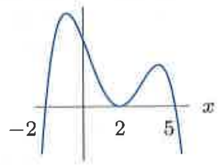


23.



In Exercises 24–26, choose the functions that are in the given family, assuming a, b, and c are constants.

$$f(x) = \sqrt{x^4 + 16} \qquad g(x) = ax^{23}$$
$$h(x) = -\frac{1}{5^{x-2}} \qquad p(x) = \frac{a^3 b^x}{c}$$
$$q(x) = \frac{ab^2}{c} \qquad r(x) = -x + b - \sqrt{cx^4}$$

24. Exponential **25.** Quadratic **26.** Linear

Problems

27. How many distinct roots can a polynomial of degree 5 have? (List all possibilities.) Sketch a possible graph for each case.

28. A rational function $y = f(x)$ is graphed in Figure 1.75. If $f(x) = g(x)/h(x)$ with $g(x)$ and $h(x)$ both quadratic functions, give possible formulas for $g(x)$ and $h(x)$.

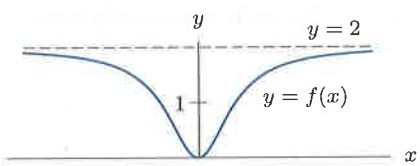


Figure 1.75

29. Find a calculator window in which the graphs of $f(x) = x^3 + 1000x^2 + 1000$ and $g(x) = x^3 - 1000x^2 - 1000$ appear indistinguishable.

30. For each function, fill in the blanks in the statements:
$f(x) \to$ __________ as $x \to -\infty$,
$f(x) \to$ __________ as $x \to +\infty$.

(a) $f(x) = 17 + 5x^2 - 12x^3 - 5x^4$
(b) $f(x) = \dfrac{3x^2 - 5x + 2}{2x^2 - 8}$
(c) $f(x) = e^x$

31. The DuBois formula relates a person's surface area s, in m^2, to weight w, in kg, and height h, in cm, by

$$s = 0.01w^{0.25}h^{0.75}.$$

(a) What is the surface area of a person who weighs 65 kg and is 160 cm tall?
(b) What is the weight of a person whose height is 180 cm and who has a surface area of 1.5 m^2?
(c) For people of fixed weight 70 kg, solve for h as a function of s. Simplify your answer.

32. According to *Car and Driver*, an Alfa Romeo going at 70 mph requires 177 feet to stop. Assuming that the stopping distance is proportional to the square of velocity, find the stopping distances required by an Alfa Romeo going at 35 mph and at 140 mph (its top speed).

33. Poiseuille's Law gives the rate of flow, R, of a gas through a cylindrical pipe in terms of the radius of the pipe, r, for a fixed drop in pressure between the two ends of the pipe.

(a) Find a formula for Poiseuille's Law, given that the rate of flow is proportional to the fourth power of the radius.
(b) If $R = 400$ cm^3/sec in a pipe of radius 3 cm for a certain gas, find a formula for the rate of flow of that gas through a pipe of radius r cm.
(c) What is the rate of flow of the same gas through a pipe with a 5 cm radius?

34. A box of fixed volume V has a square base with side length x. Write a formula for the height, h, of the box in terms of x and V. Sketch a graph of h versus x.

35. A closed cylindrical can of fixed volume V has radius r.

(a) Find the surface area, S, as a function of r.
(b) What happens to the value of S as $r \to \infty$?
(c) Sketch a graph of S against r, if $V = 10$ cm^3.

In Problems 36–38, find all horizontal and vertical asymptotes for each rational function.

36. $f(x) = \dfrac{5x - 2}{2x + 3}$ **37.** $f(x) = \dfrac{x^2 + 5x + 4}{x^2 - 4}$

38. $f(x) = \dfrac{5x^3 + 7x - 1}{x^3 - 27}$

39. The height of an object above the ground at time t is given by

$$s = v_0 t - \frac{g}{2}t^2,$$

where v_0 is the initial velocity and g is the acceleration due to gravity.

(a) At what height is the object initially?
(b) How long is the object in the air before it hits the ground?
(c) When will the object reach its maximum height?
(d) What is that maximum height?

40. A pomegranate is thrown from ground level straight up into the air at time $t = 0$ with velocity 64 feet per second. Its height at time t seconds is $f(t) = -16t^2 + 64t$. Find the time it hits the ground and the time it reaches its highest point. What is the maximum height?

41. **(a)** If $f(x) = ax^2 + bx + c$, what can you say about the values of a, b, and c if:

(i) $(1, 1)$ is on the graph of $f(x)$?
(ii) $(1, 1)$ is the vertex of the graph of $f(x)$? [Hint: The axis of symmetry is $x = -b/(2a)$.]
(iii) The y-intercept of the graph is $(0, 6)$?

(b) Find a quadratic function satisfying all three conditions.

42. A cubic polynomial with positive leading coefficient is shown in Figure 1.76 for $-10 \le x \le 10$ and $-10 \le y \le 10$. What can be concluded about the total number of zeros of this function? What can you say about the location of each of the zeros? Explain.

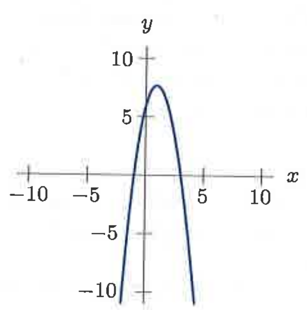


Figure 1.76

43. After running 3 miles at a speed of x mph, a man walked the next 6 miles at a speed that was 2 mph slower. Express the total time spent on the trip as a function of x. What horizontal and vertical asymptotes does the graph of this function have?

44. Which of the functions I–III meet each of the following descriptions? There may be more than one function for each description, or none at all.

(a) Horizontal asymptote of $y = 1$.
(b) The x-axis is a horizontal asymptote.
(c) Symmetric about the y-axis.
(d) An odd function.
(e) Vertical asymptotes at $x = \pm 1$.

I. $y = \dfrac{x-1}{x^2+1}$ II. $y = \dfrac{x^2-1}{x^2+1}$ III. $y = \dfrac{x^2+1}{x^2-1}$

45. Values of three functions are given in Table 1.19, rounded to two decimal places. One function is of the form $y = ab^t$, one is of the form $y = ct^2$, and one is of the form $y = kt^3$. Which function is which?

Table 1.19

t	$f(t)$	t	$g(t)$	t	$h(t)$
2.0	4.40	1.0	3.00	0.0	2.04
2.2	5.32	1.2	5.18	1.0	3.06
2.4	6.34	1.4	8.23	2.0	4.59
2.6	7.44	1.6	12.29	3.0	6.89
2.8	8.62	1.8	17.50	4.0	10.33
3.0	9.90	2.0	24.00	5.0	15.49

46. Use a graphing calculator or a computer to graph $y = x^4$ and $y = 3^x$. Determine approximate domains and ranges that give each of the graphs in Figure 1.77.

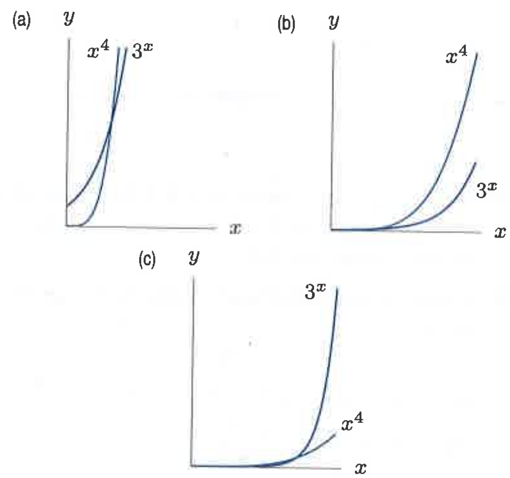


Figure 1.77

47. The rate, R, at which a population in a confined space increases is proportional to the product of the current population, P, and the difference between the carrying capacity, L, and the current population. (The carrying capacity is the maximum population the environment can sustain.)

(a) Write R as a function of P.
(b) Sketch R as a function of P.

48. Consider the point P at the intersection of the circle $x^2 + y^2 = 2a^2$ and the parabola $y = x^2/a$ in Figure 1.78. If a is increased, the point P traces out a curve. For $a > 0$, find the equation of this curve.

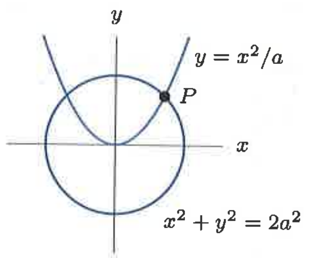


Figure 1.78

49. When an object of mass m moves with a velocity v that is small compared to the velocity of light, c, its energy is given approximately by

$$E \approx \frac{1}{2}mv^2.$$

If v is comparable in size to c, then the energy must be computed by the exact formula

$$E = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right).$$

(a) Plot a graph of both functions for E against v for $0 \le v \le 5 \cdot 10^8$ and $0 \le E \le 5 \cdot 10^{17}$. Take $m = 1$ kg and $c = 3 \cdot 10^8$ m/sec. Explain how you can predict from the exact formula the position of the vertical asymptote.

(b) What do the graphs tell you about the approximation? For what values of v does the first formula give a good approximation to E?

Strengthen Your Understanding

In Problems 50–51, explain what is wrong with the statement.

50. The graph of a polynomial of degree 5 cuts the horizontal axis five times.

51. Every rational function has a horizontal asymptote.

In Problems 52–57, give an example of:

52. A polynomial of degree 3 whose graph cuts the horizontal axis three times to the right of the origin.

53. A rational function with horizontal asymptote $y = 3$.

54. A rational function that is not a polynomial and that has no vertical asymptote.

55. A function that has a vertical asymptote at $x = -7\pi$.

56. A function that has exactly 17 vertical asymptotes.

57. A function that has a vertical asymptote which is crossed by a horizontal asymptote.

Are the statements in Problems 58–59 true or false? Give an explanation for your answer.

58. Every polynomial of even degree has a least one real zero.

59. Every polynomial of odd degree has a least one real zero.

60. List the following functions in order from smallest to largest as $x \to \infty$ (that is, as x increases without bound).

(a) $f(x) = -5x$ **(b)** $g(x) = 10^x$
(c) $h(x) = 0.9^x$ **(d)** $k(x) = x^5$
(e) $l(x) = \pi^x$

1.7 INTRODUCTION TO CONTINUITY

This section gives an intuitive introduction to the idea of *continuity*. This leads to the concept of limit and a definition of continuity in Section 1.8.

Continuity of a Function on an Interval: Graphical Viewpoint

Roughly speaking, a function is said to be *continuous* on an interval if its graph has no breaks, jumps, or holes in that interval. Continuity is important because, as we shall see, continuous functions have many desirable properties.

For example, to locate the zeros of a function, we often look for intervals where the function changes sign. In the case of the function $f(x) = 3x^3 - x^2 + 2x - 1$, for instance, we expect[38] to find a zero between 0 and 1 because $f(0) = -1$ and $f(1) = 3$. (See Figure 1.79.) To be sure that $f(x)$ has a zero there, we need to know that the graph of the function has no breaks or jumps in it. Otherwise the graph could jump across the x-axis, changing sign but not creating a zero. For example, $f(x) = 1/x$ has opposite signs at $x = -1$ and $x = 1$, but no zeros for $-1 \le x \le 1$

[38] This is due to the Intermediate Value Theorem, which is discussed on page 55.

ANSWERS TO ODD-NUMBERED PROBLEMS

Section 1.1

1 Pop 12 million in 2005

5 $y = (1/2)x + 2$

7 $y = 2x + 2$

9 Slope: $-12/7$
Vertical intercept: $2/7$

11 Slope: 2
Vertical intercept: $-2/3$

13 (a) (V)
(b) (VI)
(c) (I)
(d) (IV)
(e) (III)
(f) (II)

15 $y - c = m(x - a)$

17 $y = -\frac{1}{5}x + \frac{7}{5}$

19 Parallel: $y = m(x - a) + b$
Perpendicular:
$y = (-1/m)(x - a) + b$

21 Domain: $1 \le x \le 5$
Range: $1 \le y \le 6$

23 Domain: $0 \le x \le 5$
Range: $0 \le y \le 4$

25 Domain: all x
Range: $0 < y \le 1/2$

27 $V = kr^3$

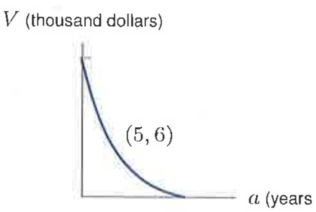
29 $S = kh^2$

31 $N = k/l^2$

33 $f(0)$ meters

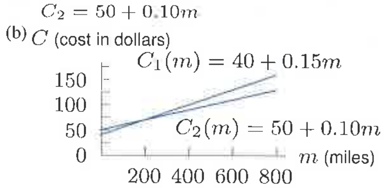
35 $f(0) = f(1) + 0.001$

37



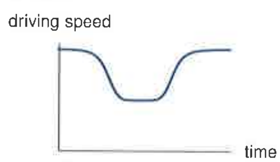
41 (a) $C = 4.16 + 0.12w$
(b) 0.12 \$/gal
(c) \$4.16

43 (a) $C_1 = 40 + 0.15m$
$C_2 = 50 + 0.10m$
(b)

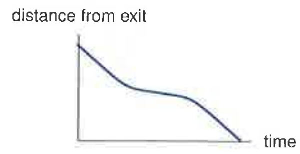


(c) For distances less than 200 miles, C_1 is cheaper.
For distances more than 200 miles, C_2 is cheaper.

45



47



49 (a) (i) $f(1985) = 13$
(ii) $f(1990) = 99$
(b) $(f(1990) - f(1985))/(1990 - 1985) =$ 17.2 billionaires/yr
(c) $f(t) = 17.2t - 34{,}129$

51 (a) 2005–2007
(b) 2004–2007

53 (a) 7.094 meters
(b) 1958, 1883

55 (a) $\Delta w/\Delta h$ constant
(b) $w = 5h - 174$; 5 lbs/in
(c) $h = 0.2w + 34.8$; 0.2 in/lb

57 (a) $C = 10 + 0.2x$
(c) Vertical intercept
Slope of line

59 (a) $(-2, 4)$
(b) $(-b, b^2)$

61 $y = 0.5 - 3x$ is decreasing

63 $y = 2x + 3$

65 False

67 False; $y = x + 1$ at points $(1, 2)$ and $(2, 3)$

69 (b), (c)

Section 1.2

1 Concave up

3 Neither

5 5; 7%

7 3.2; 3% (continuous)

9 $P = 15(1.2840)^t$; growth

11 $P = P_0(1.2214)^t$; growth

13 (a) 1.5
(b) 50%

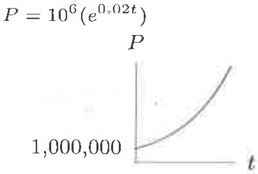
15 (a) $P = 1000 + 50t$
(b) $P = 1000(1.05)^t$

17 (a) D to E, H to I
(b) A to B, E to F
(c) C to D, G to H
(d) B to C, F to G

19 (a) $h(x) = 31 - 3x$
(b) $g(x) = 36(1.5)^x$

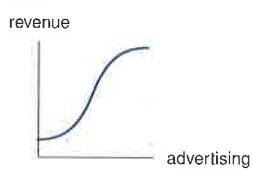
21 Table D

23 (a) $P = 10^6(e^{0.02t})$
(b)

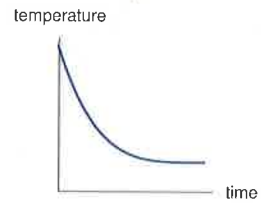


25 (a) 125%
(b) 9 times

27 (a)



(b)



29 (a) $g(x)$
(b) $h(x)$
(c) $f(x)$

31 $y = 3(2^x)$

33 $y = 2(3^x)$

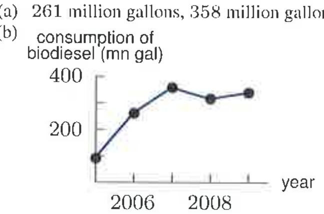
35 $f(1) = 15$, $f(3) = 25$, $f(4) = 30$
$g(1) = 10\sqrt{2}$, $g(3) = 20\sqrt{2}$, $g(4) = 40$

37 (a) 2.3 years

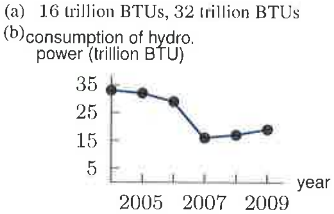
39 (a) $H, 2H, 3H$
(b) t/H; $A = 325(1/2)^{t/H}$

41 30.268%

43 (a) 261 million gallons, 358 million gallons
(b)



45 (a) 16 trillion BTUs, 32 trillion BTUs
(b)



(c) 2007, 13 trillion BTUs

47 (a) Increased: 2006, 2008; decreased: none
(b) Yes

49 $y = 2x$ not concave up

51 $f(x) = 2(1.1)^x$

53 False

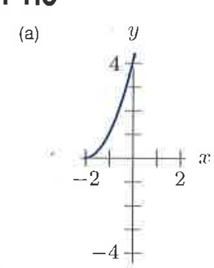
55 False

57 True; $f(x) = (0.5)^x$

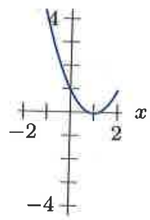
59 True

Section 1.3

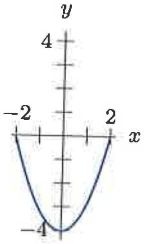
1 (a)



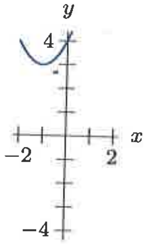
(b)



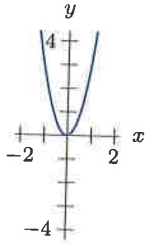
(c)



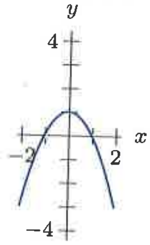
(d)



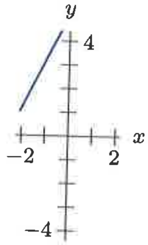
(e)



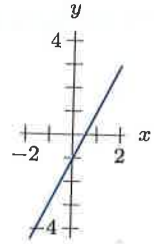
(f)



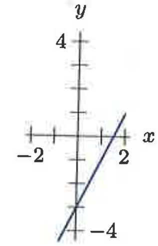
3 (a)



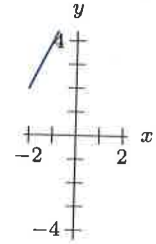
(b)



(c)



(d)



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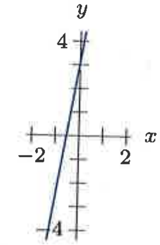

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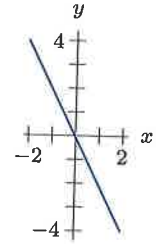

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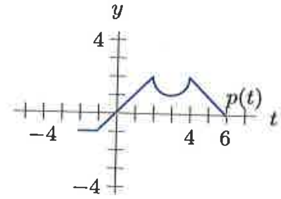

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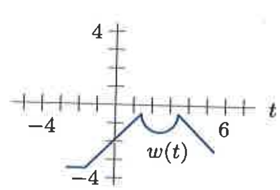


9 (a) $\sqrt{5}$
(b) 5
(c) $\sqrt{x^2+4}$
(d) $x+4$
(e) $t^2\sqrt{t+4}$

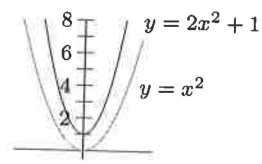
11 (a) $\frac{1}{7}$
(b) 7
(c) $\frac{1}{3x+4}$
(d) $\frac{3}{x}+4$
(e) $3+\frac{4}{t}$

13 (a) t^2+2t+2
(b) t^4+2t^2+2
(c) 5
(d) $2t^2+2$
(e) t^4+2t^2+2

15 $2zh+h^2$

17 $4hz$

21 (a) $y=2x^2+1$



(b) $y=2(x^2+1)$
(c) No

23 Not invertible

25 not invertible

27 Invertible

29 Neither

31 Neither

33 Even

35 Neither

37 $f(x)=x+1$
$g(x)=x^3$

39 $f(x)=e^x$
$g(x)=2x$

41 $y=(x-2)^3-1$

43 $\{3,-7,19,4,178,2,1\}$

45 Not invertible

47 Not invertible

49 $g(2r)$ ft^3

51 $f^{-1}(g^{-1}(10{,}000))$ min

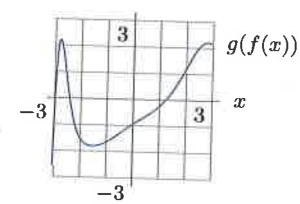
53 18

55 Cannot be done

57 0.4

59 −0.9

61



63 (a) $f(15) \approx 48$
(b) Yes
(c) $f^{-1}(120) \approx 35$
Rock is 35 millions yrs old at depth of 120 meters

65 $4000\pi/3$ cm^3

67 Reflected about t-axis, shifted up S

69 Shift left

71 $f^{-1}(x) = x$

73 $f(x) = x^2 + 2$

75 $f(x) = 1.5x$, $g(x) = 1.5x + 3$

77 True

79 False

81 True

83 True; $f(x) = 0$

85 Impossible

87 Impossible

Section 1.4

1 $1/2$

3 $5A^2$

5 $-1 + \ln A + \ln B$

7 $(\log 11)/(\log 3) = 2.2$

9 $(\log(2/5))/(\log 1.04) = -23.4$

11 1.68

13 6.212

15 0.26

17 1

19 $(\log a)/(\log b)$

21 $(\log Q - \log Q_0)/(n \log a)$

23 $\ln(a/b)$

25 $P = 15e^{0.4055t}$

27 $P = 174e^{-0.1054t}$

29 $p^{-1}(t) \approx 58.708 \log t$

31 $f^{-1}(t) = e^{t-1}$

33 16 kg

35 (a) 2023
(b) 338.65 million people

37 (a) $Q_0(1.0033)^x$
(b) 210.391 microgm/cu m

39 (a) 10 mg
(b) 18%
(c) 3.04 mg
(d) 11.60 hours

41 $C = 2$, $\alpha = -\ln 2 = -0.693$, $y(2) = 1/2$

43 (a) $B(t) = B_0 e^{0.067t}$
(b) $P(t) = P_0 e^{0.033t}$
(c) $t = 20.387$; in 2000

45 2023

47 (a) 0.00664
(b) $t = 2.167$; March 2, 2013

49 6,301 yrs; 385,081 yrs

51 (a) 47.6%
(b) 23.7%

53 2054

55 Yes

57 To the left

59 No effect

61 Function even

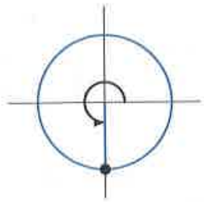
63 $f(x) = -x$

65 True

67 False

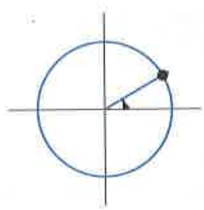
Section 1.5

1 Negative
0
Undefined

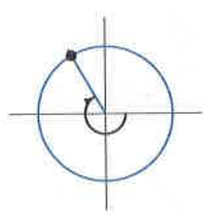


3 Positive
Positive
Positive

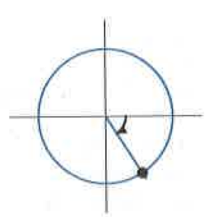
5 Positive
Positive
Positive



7 Positive
Negative
Negative



9 Negative
Positive
Negative



11 8π; 3

13 2; 0.1

15 $f(x) = 5\cos(x/3)$

17 $f(x) = -8\cos(x/10)$

19 $f(x) = 2\cos(5x)$

21 $f(x) = 3\sin(\pi x/9)$

23 $f(x) = 3 + 3\sin((\pi/4)x)$

25 0.588

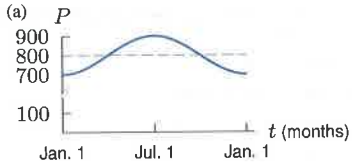
27 $(\sin^{-1}(2/5))/3 \approx 0.1372$

29 $(\tan^{-1} 2)/5 = 0.221$

31 No solution

33 If $f(x) = \sin x$ and $g(x) = x^2$ then
$\sin x^2 = f(g(x))$
$\sin^2 x = g(f(x))$
$\sin(\sin x) = f(f(x))$

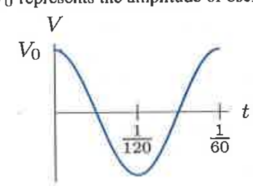
35 (a)



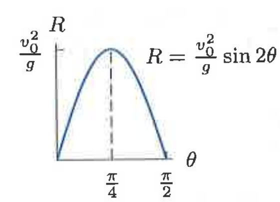
(b) $P = 800 - 100\cos(\pi t/6)$

37 (a) $f(t) = -0.5 + \sin t$
$g(t) = 1.5 + \sin t$
$h(t) = -1.5 + \sin t$
$k(t) = 0.5 + \sin t$
(b) $g(t) = 1 + k(t)$

39 (a) $\frac{1}{60}$ second
(b) V_0 represents the amplitude of oscillation.
(c)



41 $\theta = \pi/4$; $R = v_0^2/g$



43 (a) Average depth of water
(b) $A = 7.5$
(c) $B = 0.507$
(d) The time of a high tide

45 0.3 seconds

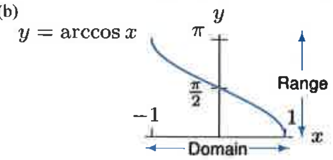
47 27.3 days $\approx$ one month

49 $f(t)$is C; $g(t)$ is B; $h(t)$ is A; $r(t)$ is D

51 (a) 2π

53 (a) 0.4 and 2.7
(b) $\arcsin(0.4) \approx 0.4$
$\pi - \arcsin(0.4) \approx 2.7$
(c) -0.4 and -2.7.
(d) $-0.4 \approx -\arcsin(0.4)$
$-2.7 \approx \arcsin(0.4) - \pi$

55 (b)



(c) The domain of arccos and arcsin are the same because their inverses (sine and cosine) have the same range
(d) $[0, \pi]$
(e) On $[-\pi/2, \pi/2]$ sine is invertible but cosine is not

57 Max $y = A + C$

59 $400(\cos x) + 1600$.

61 False

63 False

65 False

67 True

69 True

71 True

Section 1.6

1 As $x \to \infty, y \to \infty$
As $x \to -\infty, y \to -\infty$

3 $f(x) \to -\infty$ as $x \to +\infty$
$f(x) \to -\infty$ as $x \to -\infty$

5 $f(x) \to +\infty$ as $x \to +\infty$
$f(x) \to +\infty$ as $x \to -\infty$

7 $f(x) \to 3$ as $x \to +\infty$
$f(x) \to 3$ as $x \to -\infty$

9 $f(x) \to 0$ as $x \to +\infty$
$f(x) \to 0$ as $x \to -\infty$

11 $0.2x^5$

13 1.05^x

15 $25 - 40x^2 + x^3 + 3x^5$

17 (I) **(a)** 3 **(b)** Negative
(II) **(a)** 4 **(b)** Positive
(III) **(a)** 4 **(b)** Negative
(IV) **(a)** 5 **(b)** Negative
(V) **(a)** 5 **(b)** Positive

19 $y = -\frac{1}{2}(x+2)^2(x-2)$

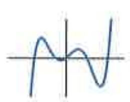
21 $f(x) = kx(x+3)(x-4)$
$(k < 0)$

23 $f(x) =$
$k(x+2)(x-2)^2(x-5)$
$(k < 0)$

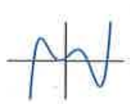
25 r

27 1, 2, 3, 4, or 5 roots

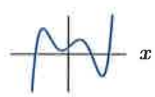
(a) 5 roots



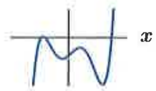
(b) 4 roots



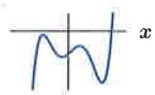
(c) 3 roots



(d) 2 roots



(e) 1 root

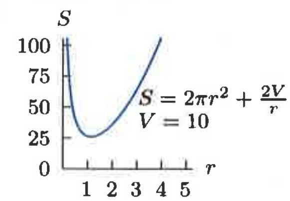


29 $-10^5 \leq x \leq 10^5, \; -10^{15} \leq y \leq 10^{15}$

31 (a) 1.3 m^2
(b) 86.8 kg
(c) $h = 112.6s^{4/3}$

33 (a) $R = kr^4$
(k is a constant)
(b) $R = 4.938r^4$
(c) $3086.42 \text{ cm}^3/\text{sec}$

35 (a) $S = 2\pi r^2 + 2V/r$
(b) $S \to \infty$ as $r \to \infty$
(c)



37 Horizontal: $y = 1$;
Vertical: $x = -2$, $x = 2$

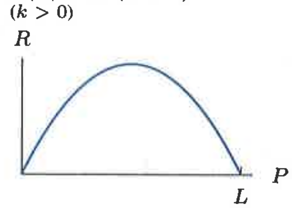
39 (a) 0
(b) $t = 2v_0/g$
(c) $t = v_0/g$
(d) $(v_0)^2/(2g)$

41 (a) (i) $1 = a + b + c$
(ii) $b = -2a$ and $c = 1 + a$
(iii) $c = 6$
(b) $y = 5x^2 - 10x + 6$

43 $(3/x) + 6/(x-2)$
Horizontal asymptote: x-axis
Vertical asymptote: $x = 0$ and $x = 2$

45 $h(t) = ab^t$
$g(t) = kt^3$
$f(t) = ct^2$

47 (a) $R(P) = kP(L - P)$
$(k > 0)$
(b)



49 (a) $v = 3 \cdot 10^8$
(b) $v < 1.5 \cdot 10^8$

51 $f(x) = (x^3 + 1)/x$ has no horizontal asymptote

53 $f(x) = 3x/(x - 10)$

55 $f(x) = 1/(x + 7\pi)$

57 $f(x) = (x-1)/(x-2)$

59 True

Section 1.7

1 Yes

3 Yes

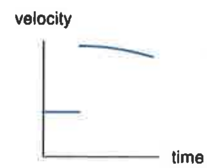
5 Yes

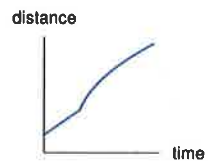
7 No

9 No

15 (a) Continuous
(b) Not continuous

17 Velocity: Not continuous
Distance: Continuous

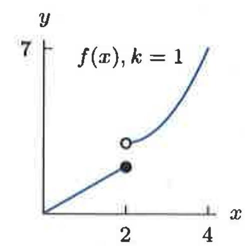




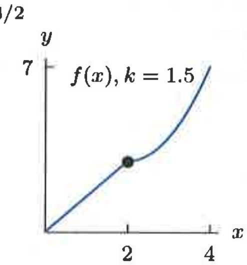
19 $k = 5/3$

21 $k = 5/4$

23 (a)



(b) 3/2
(c)



25 $k = -3$

27 $k = (\ln 3)/2$

29 $k = (e^6 - 1)/2$

31 No

33 $Q = \begin{cases} 1.2t & 0 \leq t \leq 0.5 \\ 0.6e^{0.001}e^{-.002t} & 0.5 < t \end{cases}$

35 Three zeros: one between 5 and 10, one between 10 and 12, the third either less than 5 or greater than 12